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OF THE

Cambridge Philosophical Society.

VOLUME XVIII.

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CAMBRIDGE PHILOSOPHICAL SOCIETY

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CONTENTS.

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PLATES.

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The Conductivity of Extremely Dilute Acid and Alkali Solutions. By H. H. PAINE, M.A., B.Sc., and G. T. R. EVANS, B.Sc., University College of Wales, Aberystwyth.

[*Received* 18 August 1914.]

I.

The conductivity of water distilled with the usual precautions employing block tin or Jena glass apparatus, is of the order of 1×10^{-6} reciprocal ohm (1 gemmho) at 18° C. Distilling *in vacuo*, Kohlrausch has obtained water with a conductivity as low as $0{\cdot}043 \times 10^{-6}$. This fact alone suggests that most of the contamination in the former is dissolved from the air. Direct evidence to this effect has recently been obtained by Bourdillon*, who has prepared water with a conductivity $0{\cdot}086 \times 10^{-6}$ by maintaining an atmosphere of pure air in the distilling apparatus.

This residual impurity has often been regarded as the cause of the final drop, at extreme dilution, in the equivalent conductivity curves for acid and alkali solutions. It has been shown, however, that the presence of carbonic acid alone will not account for the observed phenomenon†. It has been suggested that a substance of the nature of ammonium carbonate must be present.

Let us suppose that the distilled water contains as an impurity any substance (*e.g.* KOH or Na_2CO_3) which leads to an association of ions on the addition of a small amount of strong acid (*e.g.* H_2SO_4). The result will be that, as far as the conductivity of the solution is concerned, the first traces of the acid are used up and removed from activity—the conductivity of the new solution being less than the sum of the conductivities of the

* *Journ. Chem. Soc.*, May 1913, p. 791.

† Whetham and Paine, *Proc. Roy. Soc.*, 81 A. p. 58 (1908).

constituents taken separately. This will go on until all the impurity has been neutralised, as it were; we should then expect normal behaviour on the part of subsequent additions of H_2SO_4.

Accordingly, if we plot the *simple* conductivity (instead of the *equivalent* conductivity) of the dilute acid solution (after subtracting the conductivity of the solvent) against the concentration, we shall obtain a straight line for the region of normal behaviour (if the ionisation of the acid be complete). If this straight line be produced, it will cut the conductivity axis below the origin at a point corresponding to the loss of conductivity involved in the association of ions described above.

Whetham and Paine have already shown that this is the case for sulphuric acid solutions. The present authors have tested the rule with data supplied by Kohlrausch* and Whetham†. The straight line of Fig. 1, for example, is derived from Kohlrausch's

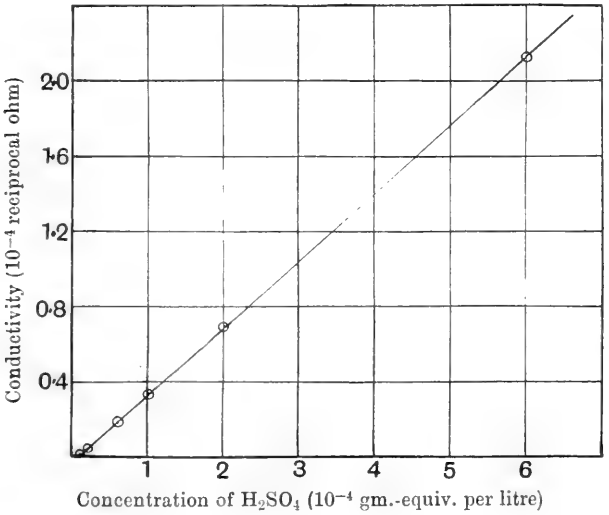

Fig. 1.

results. The experimental points lie *accurately* in a straight line for concentrations ranging from the concentration of Kohlrausch's most dilute solution up to 6×10^{-4} gm.-equivalent per litre (the neighbourhood of the maximum in the 'equivalent' conductivity curve for sulphuric acid). For greater concentrations of sulphuric acid, of course, the ionisation cannot be regarded as complete.

It may be remarked that the simple conductivity curve of a

* See Whetham, *Theory of Solutions*, p. 437.
† *Phil. Trans.*, A. 194 (1900), p. 343.

simple salt like potassium chloride is a straight line, *going through the origin.*

The form of the curve corresponding to the stage in the addition of the acid when the 'neutralisation' of the impurity is incomplete, will depend on the nature of the impurity. If it were an alkali (*e.g.* NaOH or NH_4OH) we should expect that the conductivity of the water would be reduced (the net result of the addition of the first traces of H_2SO_4 being the substitution of sulphate ions for hydroxyl ions). This reduction would continue until the neutralisation were complete, and the simple conductivity curve would immediately afterwards become normal. If the impurity were a carbonate (or carbonic acid), an actual diminution of the conductivity of the water would not be expected, since the carbonic acid molecule (H_2CO_3) is more readily ionised than the water one (H.OH), and the velocity of the carbonate ion is less than that of the hydroxyl ion. What we should expect, however, is that the simple conductivity curve would be concave (upwards) to begin with (while the carbonate ions were being suppressed), and that it would gradually become a straight line.

Exactly similar effects would result in the case of the conductivity of correspondingly dilute alkali solutions if the impurity in the water were a free acid, or the salt of a very weak base.

If the impurity in the water consisted of the salt of a strong acid or base, we should not obtain this form of association, and the form of the conductivity curve would not be influenced by any such impurity.

Whetham and Paine have already applied this hypothesis to results obtained by them, and found it to explain the facts. We shall apply it to the results obtained in the present paper. It seems to be quite sufficient for providing a quantitative explanation of the acid and alkali conductivity curves without supposing these substances to behave in a manner essentially different from that of other electrolytes.

II.

We thought it would be worth while to measure the conductivities of acid and alkali solutions more dilute than those used by Kohlrausch and Whetham,—not so much for the sake of obtaining absolute values (the simple method employed was too crude to be trustworthy to that extent) as for determining to what lower limit the simple conductivity curve for an acid or alkali solution remains a straight line, and for determining approximately what form it assumes beyond. Solutions were prepared by the additions of a stock solution from a burette, and their conductivities measured by simply transferring them to a conductivity cell.

For the purpose of measuring the conductivities of the

solutions, two conductivity cells were used. The one cell (Fig. 2) was of ordinary glass, and had been in use for some years previously for testing distilled water. Some of the earlier acid solutions

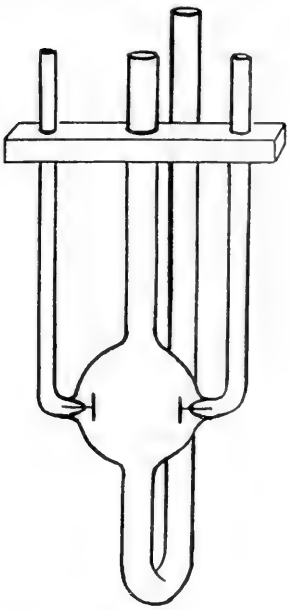

Fig. 2.

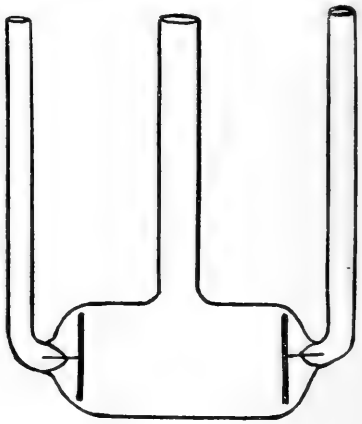

Fig. 3.

were measured in this. The second cell (Fig. 3) was made of Jena glass. Some of the acid solutions and all the alkali ones were measured in this cell. In the case of the acid solutions, the results for the two cells agreed closely, so that the solvent action on the ordinary glass walls of the former cell was of no consequence for the time the solutions remained in the cell. The electrodes in both cells had been platinised and heated to redness before being sealed into the glass. Contact with the electrodes was made by means of mercury poured into the side tubes.

The cell constants were determined by means of a standard potassium chloride solution, the conductivity of which was deduced from Kohlrausch's Tables. The resistances in all cases were measured by means of the Wheatstone's Bridge method, using the double commutator and galvanometer. The cells were maintained at 18° C. in a well-stirred water bath.

The water used for all the measurements, and also for making up the solutions, had been redistilled in a Jena glass apparatus (of 6 litre capacity) with a little potassium dichromate and sulphuric acid. The distilling flask was placed in a fume chamber,

through the side of which the condenser was brought out. Contamination from the fumes of the burner was thus completely avoided. In distilling, the first litre of water was thrown away, and the next two or three litres were collected.

Remarkably good water was obtained in this manner, the conductivity in one case being as low as $0{\cdot}36 \times 10^{-6}$ reciprocal ohm, and generally being below $0{\cdot}7 \times 10^{-6}$. This is probably due in part to the situation of the laboratory far from any large town, so that the air would contain relatively little carbonic acid and ammonia. Also the neck of the distilling flask was very long, so that spray would not reach the condenser to any great extent and the ascending column of steam would be well washed. (Compare the washing of the steam as it occurs in Bourdillon's apparatus.)

Solutions of H_2SO_4 and KOH of about milli-normal strength were prepared. Successive known quantities of these stock solutions were added to specimens of distilled water (contained in a litre Jena glass flask) by means of a burette, and the resulting concentrations calculated. The conductivities were measured after each addition by pouring samples of these solutions into the conductivity cell. From these measurements the conductivities of the solutions in reciprocal ohms were determined and were plotted against the corresponding concentrations expressed in gram-equivalents per litre.

Before giving the results, mention may be made of some of the precautions adopted to avoid contamination of the solutions during the experiments. As the solutions were extremely dilute, the possibility of contamination is naturally the first problem to be considered. The regularity of the results obtained in any one series, and the agreement between those of different series, were taken as evidence that these precautions were, on the whole, adequate.

Contamination may arise from the solution of impurities, either from the air or from the glass surfaces with which the liquid is in contact. The experiments were carried out in a well-ventilated room. Whenever liquid had to be transferred from one vessel to another, the transference was made as quickly as possible. The openings to the conductivity cells were closed by small glass caps which effectively prevented fouling from the atmosphere. The flasks (of Jena glass) were of the ordinary long-necked measuring flask pattern, and were fitted with long glass caps to keep the stoppers and lips free from dust. Before use, they were rinsed thoroughly with warm potash and concentrated nitric acid; they were afterwards steamed for several hours; finally they were filled with distilled water and allowed to stand for several days. After this treatment they were inverted in distilled water with the stoppers loose, so that the lips and stoppers should be

thoroughly cleaned; the water in which they stood was renewed twice a day for three days. The glass caps were treated in the same manner.

Again, when transferring a solution from one vessel to another, a small quantity was always poured away first of all, for the purpose of washing that part of the lip over which the liquid flows.

In order to determine what sort of error was possible through the transference of such extremely dilute solutions to the conductivity cell, three or four samples of water were taken successively from the same flask, and their resistances measured. Series of readings of this nature were made from time to time with different specimens of distilled water. The following four series will serve as examples. The readings are in ohms.

I	II	III	IV
710,000	690,000	541,000	1,570,000
664,000	650,000	592,000	1,510,000
717,000	665,000	640,500	1,500,000
		592,000	1,600,000

These resistances show no continuous decrease. Variations in the readings certainly occur, but the fluctuations are irregular, and there is no distinct evidence that the water in the flask was fouled through the opening of the flask on these occasions. The fouling, where it occurs, is rather in that portion of the water that flows out into the conductivity cell.

Errors of this nature, it will be noticed, would have the effect of introducing irregularity amongst the individual readings of resistances, but would not appreciably alter the general direction of the conductivity curve plotted in the manner already described.

There was an effect other than fouling, however, which proved of far more serious consequence, namely, what seemed to be adsorption on the surfaces of the glass vessels. For these very dilute solutions this could not be ignored. In the case of the acid solutions it was apparently negligible, but in that of the alkali solutions it rendered the results of little value. Thus the conductivity of the very dilute alkali solutions gradually diminished, while the conductivity of pure water subsequently introduced into the vessel gradually increased. Consequently only the sulphuric acid results are recorded.

III.

The following, Table I, is a sample of the series of experiments performed. It contains the measurements made on a series of sulphuric acid solutions prepared with one specimen of distilled water. The cell constant $(k) = 0{\cdot}550$.

TABLE I. *Sulphuric Acid.*

Concentration of Solution (gm.-equiv. per litre)	Resistance of Solution in Cell (ohms) ($=R$)	Conductivity of Solution $\left(=\frac{k}{R}\right)$
—	1,545,000	$3{\cdot}6\times 10^{-7}$
$2{\cdot}9\times 10^{-7}$	1,400,000	3·9
10·7	1,010,000	5·5
19·2	735,000	7·5
28·4	600,000	9·2
38·4	485,000	11·3
49·4	377,500	14·6
61·5	317,000	17·4
88·3	210,500	26·1
149·5	112,500	48·9
218	75,860	72·5
298	54,100	101·7
394	40,660	135·3

Fig. 4 shows these results plotted. The residual conductivity of the water has *not* been deducted from the conductivity readings in this case.

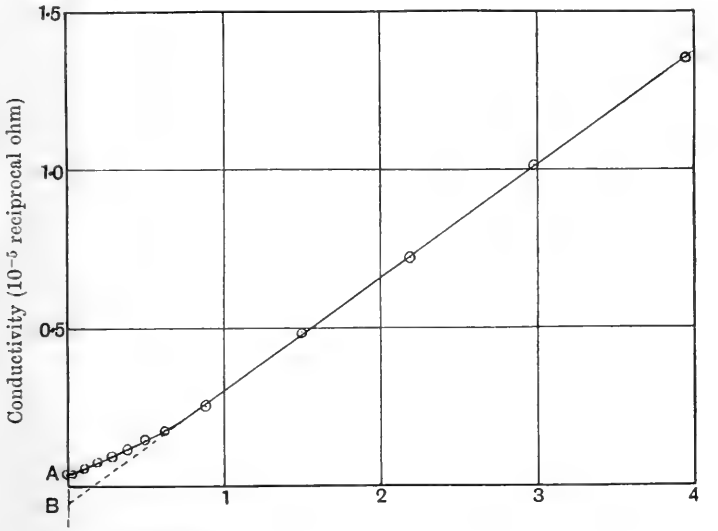

Fig. 4.

It will be noticed that the straight line portion of the curve, when produced, cuts the conductivity axis at a point B, which is below the starting point A. The portion AB represents what we may call the 'loss of conductivity,' the net result of the initial irregularity in the simple conductivity curve.

Table II is a summary of the observations made with four specimens of distilled water. The units are the same as before.

TABLE II.

No. of Series	Initial conductivity of water	Lower concentration at which line ceases to be straight	'Loss of Conductivity' $= AB$ in Fig. 4	dk/dm the tangent of the straight line
1	$0{\cdot}61 \times 10^{-6}$	$1{\cdot}0 \times 10^{-5}$	$1{\cdot}2 \times 10^{-6}$	0·369
2	$0{\cdot}64 \times 10^{-6}$	$0{\cdot}7 \times 10^{-5}$	$0{\cdot}7 \times 10^{-6}$	·362
3	$0{\cdot}36 \times 10^{-6}$	$0{\cdot}7 \times 10^{-5}$	$0{\cdot}8 \times 10^{-6}$	·355
4	$0{\cdot}53 \times 10^{-6}$	$0{\cdot}8 \times 10^{-5}$	$0{\cdot}5 \times 10^{-6}$	·364
			Mean ...	0·362

IV.

Let us now examine these results in the light of the hypothesis mentioned in the first section of this paper, dealing in turn with the last three columns of Table II.

(1) In the first place it will be seen that the straight line persists for concentrations a little smaller than the smallest of those examined by Kohlrausch (10^{-5} gm.-equiv. per litre). That is to say, the irregularities cease when the quantity of acid added amounts to about $0{\cdot}8 \times 10^{-5}$ gm.-equiv. per litre, the residual impurities of the distilled water being then 'neutralised.'

(2) Again, a comparison of the numbers in columns 2 and 4 of Table II will show that the 'loss of conductivity' in any case is generally greater than the initial conductivity of the water. This is interesting, for it is the easiest way of seeing that the presence of carbonic acid alone could not, as a rule, account for the initial irregularities. For any loss of conductivity that occurred would be due simply to the association of this (ionised) carbonic acid through the addition of a strong acid. Consequently the maximum loss of conductivity could not be greater than the initial conductivity of the solvent. On the other hand, with an alkali or a carbonate, say ammonium carbonate, as the residual

impurity of the water, the association which occurred through the addition of a strong acid would involve the disappearance of some of the hydrogen ions added, and not of the ammonium ions originally present. The loss of conductivity would consequently be greater than the original conductivity of the solvent, inasmuch as the velocity of the hydrogen ion is greater than that of the ammonium ion.

For example, in Fig. 4 the loss of conductivity AB is about $0{\cdot}8 \times 10^{-6}$. This is due, we shall suppose, to the combination of carbonate ions (from the solvent) with hydrogen ions (from the acid added). The amount of (ionised) ammonium carbonate required to bring about this loss of conductivity on the addition of strong acid would itself possess a conductivity of

$$\frac{70 + 50}{320 + 50} \times 0{\cdot}8 \times 10^{-6} = 0{\cdot}3 \times 10^{-6} \text{ reciprocal ohm,}$$

(where the numbers 320, 70 and 50 are the velocity numbers of the H, NH_4 and $\frac{1}{2}$ CO_3 ions respectively). The initial conductivity of the water is of this order.

It will be seen, therefore, that the loss of conductivity in any particular instance is readily accounted for *quantitatively* on the assumption that the residual conductivity of the water is due to the absorption of ammonia and carbonic acid from the air in suitable proportions. The proportion would naturally vary with the actual conditions attending each distillation, just as the conductivity of the water itself varies. The important point is that the quantities which the acid curve makes it necessary to assume are always such as can be fitted in with the conductivity of the solvent itself.

(3) There is another important conclusion to be noticed. These results provide us with a method of deducing the 'true value' of k/m, the equivalent conductivity, for these solutions at infinite dilution. The straight line indicates that ionisation is complete (within the limits of observation) for the concentrations included. Hence the value of dk/dm, the tangent of this straight line, is the limiting value of k/m for 'infinite dilution.' These quantities are given in the last column of Table II. In a similar way, using Kohlrausch's results (Fig. 1), we obtain $dk/dm = 0{\cdot}364$. The maximum value of k/m in the *equivalent* conductivity curve of Kohlrausch is 0·355.

(4) An illustrative series of experiments was carried out with a very dilute solution of ammonium carbonate. Successive quantities of sulphuric acid were added in the same manner as for the previous experiments, and the conductivities of the solutions were measured. The initial solution contained about 7×10^{-5} gram-molecule of pure $(NH_4)_2CO_3$ per litre, and had a conductivity of

$9{\cdot}0 \times 10^{-6}$ reciprocal ohm. The curve obtained by plotting the results is shown in Fig. 5. It is exactly similar to those obtained with the pure water, only the phenomena we are studying are shown on a larger scale. Also, dk/dm for the 'normal' part of the curve is 0·373.

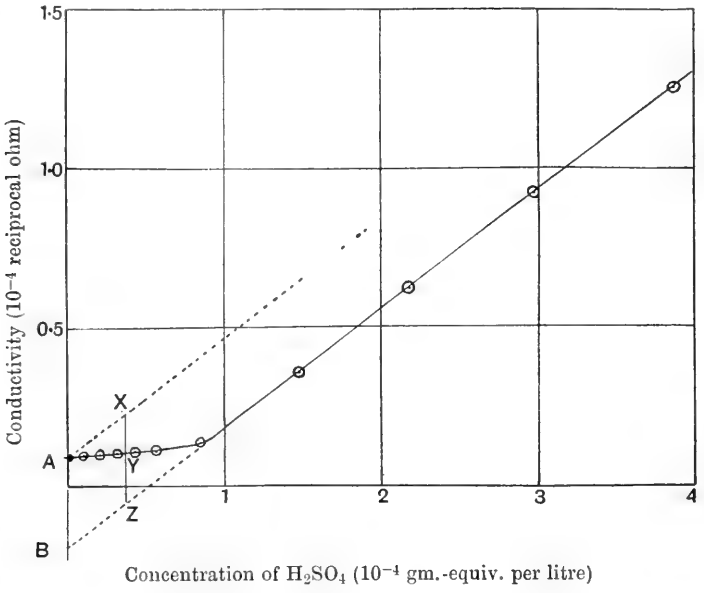

Fig. 5.

(5) We have supposed that ammonia and carbonic acid gas are dissolved in the distilled water from the air, and that on the addition of a strong acid, the hydrogen and carbonate ions combine to form undissociated carbonic acid. Let us suppose that this takes place according to the equation

$$2H^+ + CO_3^{--} = H_2CO_3*.$$

Let m_1 and m_2 represent, respectively, the number of gram-equivalents of H^+ and CO_3^{--} ions present in one litre, and m_3 the number of gram-equivalents of undissociated H_2CO_3. Then when equilibrium is attained we have

$$m_1^2 m_2 = p \,.\, m_3$$

or

$$\frac{m_3}{m_2} = \frac{m_1^2}{p},$$

* See Note at the end of the Section.

where p is a constant. Hence, for any given value of the *ratio* $\frac{m_3}{m_2}$ we should have only one value of m_1 wherever the reaction occurs.

Now in the illustrative experiments with the ammonium carbonate solutions, the form of the curve certainly depended on this reaction. The value of m_1 as calculated from Fig. 5 for the ammonium carbonate solution is found to be the same as that calculated from Fig. 4 for the distilled water.

Thus in Fig. 5, if A be the initial conductivity of the solution, the straight line drawn through A would represent the conductivity of the acid solution if no combination of ions had taken place. Take any vertical line XYZ as shown. XY is the 'loss of conductivity' at Y. XZ is the total loss of conductivity that ultimately takes place, so that YZ is proportional to the number of carbonate ions still in solution.

Hence $$\frac{m_3}{m_2} = \frac{XY}{YZ},$$

since m_3 and m_2 are expressed in gram-equivalents. Also m_1 is the quantity of hydrogen ions present, *i.e.* the amount added in the sulphuric acid (represented by the abscissa of the point Y) minus the amount removed as unionised H_2CO_3 (proportional to XY). For ionised H_2SO_4, $k/m = 0{\cdot}362$. Hence, since the velocity numbers of the H, $\frac{1}{2}SO_4$ and $\frac{1}{2}CO_3$ ions are 320, 73 and 50 respectively, the value of k/m for ionised H_2CO_3

$$= 0{\cdot}362 \times \frac{370}{393} = 0{\cdot}340.$$

Hence XY corresponds to the removal of $\frac{XY}{0{\cdot}340}$ gram-equivalents of H_2CO_3 (and therefore of H ions) from the ionised state.

We have worked out the case in which $\frac{XY}{YZ} = \frac{m_3}{m_2} = 1$, calculating the values of m_1 from the Figs. 4 and 5.

(*a*) For the distilled water (Fig. 4),

$$XY = YZ = 0{\cdot}44 \times 10^{-6} \text{ reciprocal ohm,}$$

which corresponds to $0{\cdot}44 \times 10^{-6} \div 0{\cdot}340 = 1{\cdot}3 \times 10^{-6}$ gr.-equiv. of H_2CO_3 (and of the corresponding H ions) per litre. This occurs at a point of the curve for which the abscissa is $2{\cdot}2 \times 10^{-6}$ gr.-equiv. of H ions added in the H_2SO_4. Hence the quantity of H ions still in the solution is the difference of these, viz. $0{\cdot}9 \times 10^{-6}$ gr.-equiv. per litre ($= m_1$).

(*b*) For the ammonium carbonate solution (Fig. 5),

$$XY = YZ = 14{\cdot}5 \times 10^{-6} \text{ reciprocal ohm,}$$

which corresponds to $14{\cdot}5 \times 10^{-6} \div 0{\cdot}340 = 42{\cdot}7 \times 10^{-6}$ gr.-equiv. of H_2CO_3 (and of the corresponding H ions) per litre. This occurs at a point of the curve for which the abscissa is $43{\cdot}4 \times 10^{-6}$ gr.-equiv. of H ions added in the H_2SO_4. Hence the quantity of H ions still in solution as such is the difference of these, viz. $0{\cdot}7 \times 10^{-6}$ gr.-equiv. per litre ($= m_1$)*.

The fact that these two values of m_1 are the same is very strong evidence that we are dealing with the same chemical or physical change in both cases, that is to say, that the initial irregularity in the sulphuric acid curve is due to this particular action proceeding.

V.

These experiments and calculations point with considerable certainty to the conclusion that the impurity constituting the main part of the residual conductivity of ordinary distilled water provides a sufficient explanation of the irregularity observed with extremely dilute acid solutions. They also point to the conclusion that this impurity consists mainly of a carbonate.

Similar measurements on alkali solutions should throw some light on the nature of this carbonate. As already mentioned,

* The recombination of the ions of carbonic acid is probably more complicated than what we have assumed above, taking place in two stages,

$$H^+ + CO_3^{--} = H\,CO_3^-,$$

and

$$H^+ + HCO_3^- = H_2CO_3.$$

Let m_1, m_2, and m_3 represent the same quantities as before, and let m_4 represent the number of gram-equivalents of HCO_3 ions present. Then we have for equilibrium

$$m_1 m_2 = p_1 . m_4,$$

and

$$m_1 m_4 = p_2 . m_3,$$

where p_1 and p_2 are constants;

whence

$$\frac{m_3}{m_2} = \frac{m_1^2}{p_1 p_2},$$

an equation identical with the previous one.

As for the conductivities, assuming the velocity number of the HCO_3 ion to be about the same as that of the $\frac{1}{2}CO_3$ ion (which is probable) the 'loss of conductivity' at Y ($= XY$) will be that due to the disappearance of $(m_3 + m_4)$ gr.-equiv. of H and CO_3 ions, and the loss of conductivity still possible ($= YZ$) will be the conductivity of $(m_2 + m_4)$ gr.-equiv. of H and CO_3 ions. Hence $\frac{XY}{YZ} = \frac{m_3 + m_4}{m_2 + m_4}$, and where this ratio is unity (for which case the above numerical calculations are made), we have again $\frac{XY}{YZ} = \frac{m_3}{m_2} = 1$. Also, $(m_3 + m_4)$ gr.-equiv. of H ions have disappeared, *i.e.* $\frac{XY}{0{\cdot}340}$ gr.-equiv. The calculation we have made on the simpler assumption still holds therefore. (Note that the total loss of conductivity possible is due to the disappearance of $(m_2 + m_3 + 2m_4)$ gr.-equiv. of H_2CO_3 from the ionised state, m_4 gr.-equiv. of HCO_3 ions giving rise to $2m_4$ gr.-equiv. of H_2CO_3.)

these experiments are much more uncertain on account of the danger of adsorption by the glass vessels used. For obtaining any trustworthy results, it would be necessary to employ vessels of a material that does not behave in this way. For the present, therefore, we shall mention merely general results.

Alkali solutions show a drop in the k/m curves similar to that found for acids, and we should therefore expect a similar explanation. Hence the suggestion that the kation we are dealing with is ammonium, since unionised $NH_4.OH$ would be produced on the addition of a strong alkali, just as unionised H_2CO_3 is produced on the addition of a strong acid. If there be an excess of free carbonic acid in the distilled water, the 'loss of conductivity' is still more obvious. The fact that both NH_3 and CO_2 are normal constituents of the atmosphere lends a good deal of support to the suggestion.

Again, if a simple conductivity curve be drawn with Kohlrausch's figures for KOH, the result is very nearly a straight line, but not as accurate a one as the corresponding H_2SO_4 curve. It appears to be approaching concavity at greater concentrations than in the case for the acid. On the ammonium carbonate hypothesis, this is indeed what we should expect; for though NH_4OH is a weak base, it is more easily ionised than H_2CO_3; so that we cannot expect the bending away from the straight line at the foot of the conductivity curve to be so sudden; it should begin at greater concentrations and proceed more gradually.

The hypothesis we have been developing demands that if we could work with water of the degree of purity attained by Kohlrausch, preventing the CO_2 and NH_3 of the air from reaching the water at any stage of the experiments, the drop in the k/m curve would be pushed into a region of much smaller concentration,—or, more exactly, the 'loss of conductivity' would be much reduced. In Kohlrausch's experiments (Fig. 1), this loss is $2{\cdot}7 \times 10^{-6}$ reciprocal ohm; in our experiments (Table II) it is about $0{\cdot}8 \times 10^{-6}$. It could never be made to disappear entirely however, on account of the real ionisation of the water itself, and of the recombination of the H and OH ions on the addition of an acid.

Studies in Synthetic Logic. By NORBERT WIENER, Ph.D. (Communicated by Mr G. H. Hardy.)

[*Received* 13 July 1914.]

§ 1. In a recent article of mine in the *Proceedings of the Cambridge Philosophical Society**, I showed how we can regard the series of the instants of time as a construction from the non-serial relation of complete temporal succession between events in time, and how only a few simple presuppositions concerning the formal character of this relation of complete temporal succession sufficed to establish the seriality of the relation of succession between instants; and, in a foot-note, I showed further how, *without making any assumptions* concerning the formal properties of a given relation, P, we can construct another relation from P in a perfectly determinate manner, so that this latter relation will always be a series.

In this article, I wish to extend this method of series-construction in two different directions. I first mean to bring the definitions of order through triadic and tetradic relations under a single very general heading, and to show that Frege's theory of hereditary relations and the theory of series-synthesis developed in my former article can be generalized so as to apply to these. Then I shall give an alternative method of constructing series from non-serial relations which bears much the same relation to the various series of sensation-*intensities* that the method of my previous article bears to the series of instants that constitutes one sort of *extension*, time.

In general, our symbolism will be that of the *Principia Mathematica* of Whitehead and Russell, and we shall take the theorems established in that book for granted. But as we shall have much to do with polyadic relations, and as the parts of the *Principia* which will treat of general polyadic relations are not yet in print, it will be necessary for us to develop a symbolism of our own here. Such properties of polyadic relations as have precise analogues in the theory of classes we shall take for granted. Moreover, as we shall want to speak of properties of relations among *any* number of terms, and as in Mr Russell's system†, relations among m terms belong to different types than relations among n terms, if $m \neq n$, so that no propositional functions whose arguments range over

* "A Contribution to the Theory of Relative Position," vol. XVII, Part 5, pp. 441—9.

† See, however, my article, "A Simplification in the Logic of Relations," *Proc. Camb. Phil. Soc.*, vol. XVII, Part 5, pp. 387—90. The method of this article can be extended to n-adic relations in general.

m-adic and n-adic relations exist, we shall have to permit a certain logical laxity in our symbolism. Though our theorems really demand a separate, though precisely parallel, proof when the relations dealt with are m-adic and when they are n-adic, we shall have to treat these proofs as one. Though every relation holds among a definite set of terms, we shall permit dots to fill the places of an indefinite number of these. Though the analogues of $\dot{\cap}$, $\cup$, $\dot{s}$, etc. are different with each different sort of relation with which they have to do, we shall represent them all by the symbols we use in the case of binary relations. To the reader acquainted with symbolic logic, there will be no difficulty in reducing any particular case of the theorems I prove to a strictly rigorous form.

§ 2. Let us write the proposition, '$a_1, a_2, \ldots, a_n$ are in the n-adic relation R,' as $R\{a_1, a_2, \ldots, a_n\}$. I shall call a property of an n-adic relation, R, an *n-transitivity* of R when it can be written in the form

$$(1)\quad (\exists b_1, b_2, \ldots, b_k) \,.\, T_R\{a_1, a_2, a_3, \ldots, a_n, b_1, b_2, \ldots, b_k\} \,.\, \supset_{a_1, a_2, \ldots, a_n} \,.\, R\{a_1, a_2, \ldots, a_n\},$$

where T_R is the logical disjunction of a number of expressions in the form

$$R\{c_1, c_2, \ldots, c_n\} \,.\, R\{c_1', c_2', \ldots, c_n'\} \,.\, R\{c_1'', c_2'', \ldots, c_n''\} \ldots R\{c_1^{(l)}, c_2^{(l)}, \ldots, c_n^{(l)}\},$$

where l is not necessarily the same in each of these expressions, and $c_1, c_2, \ldots, c_n, c_1', c_2', \ldots, c_n', \ldots, c_1^{(l)}, c_2^{(l)}, \ldots, c_n^{(l)}$, which are not all distinct from one another, are to be found among $a_1, a_2, \ldots, a_n, b_1, b_2, \ldots, b_k$. Ordinary binary transitivity is an example of a 2-transitivity; the property of 'betweenness,' which may be written

$$(\exists d) : abd \,.\, bdc \,.\, \vee \,.\, abd \,.\, bcd \,.\, \vee \,.\, adc \,.\, dbc \,.\, \vee \,.\, abd \,.\, acd \,.\, bac \,.\, \vee \,.\, dab \,.\, dac \,.\, bac \,.\, \vee \,.\, bca : \supset_{a,b,c} \,.\, abc,$$

is a 3-transitivity; the property of Vailati's separation-relation, which may be written

$$(\exists e) : ab \,\|\, dc \,.\, \vee \,.\, cd \,\|\, ab \,.\, \vee \,.\, ab \,\|\, ec \,.\, ae \,\|\, cd : \supset_{a,b,c,d} \,.\, ab \,\|\, cd,$$

is a 4-transitivity. From these examples it is obvious that the transitivity-properties of relations are of very great logical interest, and that a method which shall point out significant analogies between the various sorts of transitivity is not without importance.

One property which all sorts of n-transitivity have in common is this: if R is any n-adic relation whatever, then it is always possible, given any particular form of n-transitivity, to construct in a perfectly determinate manner a relation, R',

including R, forming a well-defined function of R, having the desired sort of transitivity.

This is proved as follows: let the n-transitivity in question be the one given in (1). Decompose $T_R\{a_1, \dots, a_n, b_1, \dots, b_k\}$, as indicated, into a sum of expressions of the form

$$R\{c_1, c_2, \dots, c_n\} \cdot R\{c_1', c_2', \dots, c_n'\} \dots R\{c_1^{(l)}, c_2^{(l)}, \dots, c_n^{(l)}\}.$$

Let there be, say, f such expressions, the pth one always with l_p R's. Replace each of these R's by one and one only of the variable relations $X_1, X_2, \dots, X_m$, with the same arguments as the R it replaces, and let $m = \sum_{p=1}^{p=f} l_p$. We shall thus transform T_R into a relation which is a function of the m variable relations $X_1, X_2, \dots, X_m$. Let us call this relation $\overset{T}{X_1 X_2 \dots X_m}$. Now, let us define the relation $\underset{T}{X_1 X_2 \dots X_m}$ as follows:

$$(2)\quad \underset{T}{X_1 X_2 \dots X_m}\{a_1, a_2, \dots, a_n\} \cdot = \cdot (\exists b_1, b_2, \dots, b_k) \cdot \overset{T}{X_1 X_2 \dots X_m}\{a_1, a_2, \dots, a_n, b_1, b_2, \dots, b_k\} \quad \text{Df.}$$

Like $\overset{T}{X_1 X_2 \dots X_m}$, $\underset{T}{X_1 X_2 \dots X_m}$ is a function of $X_1, X_2, \dots, X_m$, where the latter may assume any values which are n-adic relations. Now, I define the class of T-powers of R, or, as I write it, $\overrightarrow{T}_{\mathrm{pr}}{}'R$, as follows:

$$(3)\quad T_{\mathrm{pr}} = \hat{S}\hat{R}\{X_1, X_2, \dots, X_m \in \mu \cdot \supset_{X_1, X_2, \dots, X_m} \cdot \underset{T}{X_1 X_2 \dots X_m} \in \mu : R \in \mu : \supset_\mu \cdot S \in \mu\} \quad \text{Df.}$$

I make the further definition,

$$(4)\quad R_T = \dot{s}'\overrightarrow{T}_{\mathrm{pr}}{}'R \quad \text{Df.}$$

Now, R_T includes R and is a function of it, and has the desired sort of n-transitivity.

First, R_T includes R. For, since, as may be seen on inspection, $RT_{\mathrm{pr}}R, R \in \overrightarrow{T}_{\mathrm{pr}}{}'R$. Since every member of a class is included in the sum of the class, $R \subseteq \dot{s}'\overrightarrow{T}_{\mathrm{pr}}{}'R \subseteq R_T$. Secondly, as R_T is derived from R by a process which is really perfectly definite (though I admit that some of the stages of the process by which I have derived R_T from R are not uniquely determined, a little reflection will convince one that all the possible determinations of $\overset{T}{X_1 X_2 \dots X_m}$ yield the same value of R_T), it is a function of R, and

of R alone, once T is determined. Thirdly, R_T has the desired sort of n-transitivity. For we can write

$$T_{R_T}\{a_1, a_2, \dots, a_n, b_1, b_2, \dots, b_k\}$$

as a sum of products of the form

$$R_T\{c_1, c_2, \dots, c_n\} \cdot R_T\{c_1', c_2', \dots, c_n'\} \dots R_T\{c_1^{(l)}, c_2^{(l)}, \dots, c_n^{(l)}\}.$$

Now to say $R_T\{d_1, d_2, \dots, d_n\}$ is, by the definition of R_T, the same as to say that there is some S such that $ST_{\mathrm{pr}}R$, and $S\{d_1, d_2, \dots, d_n\}$. Therefore

$$T_{R_T}\{a_1, a_2, \dots, a_n, b_1, b_2, \dots, b_k\}$$

is equivalent to

$$(\exists S_1, S_2, \dots, S_m) \cdot \underbrace{\overset{T}{S_1 S_2 \dots S_m}}\{a_1, a_2, \dots, a_n, b_1, b_2, \dots, b_k\} \cdot S_1 T_{\mathrm{pr}} R \cdot S_2 T_{\mathrm{pr}} R \cdot S_3 T_{\mathrm{pr}} R \dots S_m T_{\mathrm{pr}} R.$$

Therefore

$$(5)\quad \vdash :: . (\exists b_1, b_2, \dots, b_k) \cdot T_{R_T}\{a_1, a_2, \dots, a_n, b_1, b_2, \dots, b_k\} ::$$

$$\equiv :: (\exists S_1, S_2, \dots, S_m) \cdot \underbrace{S_1 S_2 \dots S_m}_{T}\{a_1, a_2, \dots, a_n\} \cdot S_1 T_{\mathrm{pr}} R \cdot S_2 T_{\mathrm{pr}} R \cdot S_3 T_{\mathrm{pr}} R \dots S_m T_{\mathrm{pr}} R ::$$

$$\equiv :: (\exists S_1, S_2, \dots, S_m) : . \underbrace{S_1 S_2 \dots S_m}_{T}\{a_1, a_2, \dots, a_n\} : . X_1, X_2, \dots, X_m \in \mu . \supset_{X_1, X_2, \dots, X_m} . \underbrace{X_1 X_2 \dots X_m}_{T} \in \mu : R \in \mu : \supset_\mu . S_1, S_2, \dots, S_m \in \mu ::$$

$$\supset :: (\exists S_1, S_2, \dots, S_m) : . \underbrace{S_1 S_2 \dots S_m}_{T}\{a_1, a_2, \dots, a_n\} : . X_1, X_2, \dots, X_m \in \mu . \supset_{X_1, X_2, \dots, X_m} . \underbrace{X_1 X_2 \dots X_m}_{T} \in \mu : R \in \mu : \supset_\mu . \underbrace{S_1 S_2 \dots S_m}_{T} \in \mu ::$$

$$\supset :: (\exists S_1, S_2, \dots, S_m) \cdot \underbrace{S_1 S_2 \dots S_m}_{T}\{a_1, a_2, \dots, a_n\} \cdot \underbrace{S_1 S_2 \dots S_m}_{T} T_{\mathrm{pr}} R ::$$

$$\supset :: (\exists U) \cdot U\{a_1, a_2, \dots, a_n\} \cdot U \in \overrightarrow{T}_{\mathrm{pr}}{}'R ::$$

$$\supset :: (\hat{s}'\overrightarrow{T}_{\mathrm{pr}}{}'R)\{a_1, a_2, \dots, a_n\} ::$$

$$\supset :: R_T\{a_1, a_2, \dots, a_n\}.$$

This is what we wished to prove, for, if we compare this with (1), it shows that R_T has the desired sort of transitivity.

When the transitivity in question is ordinary binary transitivity R_T becomes R_{po}. In general, the appropriate form of R_T performs the function of R_{po} in systems whose order is given by a triadic or tetradic or other polyadic relation.

§ 3. There is another important sort of property which the ordinary serial relation, the 'between' relation on a given line, and the separation-relation have in common. For the binary serial relation, it is ordinary connexity; for the 'between' relation on a given line it may be expressed in symbols as

$$(\exists m, n) : amn \,.\, \vee \,.\, man \,.\, \vee \,.\, mna : bmn \,.\, \vee \,.\, mbn \,.\, \vee \,.\, mnb :$$
$$cmn \,.\, \vee \,.\, mcn \,.\, \vee \,.\, mnc : \supset_{a,b,c} :$$
$$a = b \,.\, \vee \,.\, b = c \,.\, \vee \,.\, c = a \,.\, \vee \,.\, abc \,.\, \vee \,.\, bca \,.\, \vee \,.\, cab ;$$

for the separation-relation it is

$$(\exists m, n, o) : am \parallel no \,.\, \vee \,.\, ma \parallel no \,.\, \vee \,.\, mn \parallel ao \,.\, \vee \,.\, mn \parallel oa :$$
$$bm \parallel no \,.\, \vee \,.\, mb \parallel no \,.\, \vee \,.\, mn \parallel bo \,.\, \vee \,.\, mn \parallel ob :$$
$$cm \parallel no \,.\, \vee \,.\, mc \parallel no \,.\, \vee \,.\, mn \parallel co \,.\, \vee \,.\, mn \parallel oc :$$
$$dm \parallel no \,.\, \vee \,.\, md \parallel no \,.\, \vee \,.\, mn \parallel do \,.\, \vee \,.\, mn \parallel od :$$
$$\supset_{a,b,c,d} : a = b \,.\, \vee . b = c \,.\, \vee \,.\, c = d . \vee . d = a \,.\, \vee . a = c \,.\, \vee . b = d \,.\, \vee .$$
$$ab \parallel cd \,.\, \vee \,.\, ac \parallel bd \,.\, \vee \,.\, ad \parallel bc.$$

For the sake of brevity, let us generalize the notion of 'field' in the following manner:

$$(6) \quad C = \hat{\alpha}\hat{R}\{\alpha = \hat{x}\{(\exists a_1, a_2, \ldots, a_{n-1})\} :$$
$$R\{x, a_1, a_2, \ldots, a_{n-1}\} \,.\, \vee \,.\, R\{a_1, x, a_2, \ldots, a_{n-1}\} \,.\, \vee \ldots$$
$$\vee \,.\, R\{a_1, a_2, \ldots, a_{n-1}, x\}\} \quad \text{Df.}$$

Now, I shall define a property of an n-adic relation, R, as an *n-connexity* of that relation if it can be written in the form

$$(7) \quad a_1, a_2, \ldots, a_n \in C'R : l \neq m \,.\, \supset_{l,m} : \sim(a_l \neq a_m) : \supset_{a_1, a_2, \ldots, a_n} :$$
$$R\{a_1, a_2, \ldots, a_n\} \,.\, \vee \,.\, R\{a_1', a_2', \ldots, a_n'\} \,.\, \vee \,.$$
$$R\{a_1'', a_2'', \ldots, a_n''\} \,.\, \vee \ldots \vee \,.\, R\{a_1^{(p)}, a_2^{(p)}, \ldots, a_n^{(p)}\},$$

where $a_1' \ldots a_n', a_1'' \ldots a_n'', \ldots, a_1^{(p)} \ldots a_n^{(p)}$ are each definite permutations of $a_1 \ldots a_n$. It is obvious that ordinary binary connexity is, by this definition, a 2-connexity, and that the properties of 'between' and separation which we have just mentioned are, respectively, 3- and 4-connexities.

Now, I wish to raise with regard to n-connexities the precise analogue of the question which we raised with regard to n-transitivities in the last section: is it possible, given any n-adic relation and any n-connexity, to form by a perfectly definite method an n-adic relation genuinely dependent on this relation, having the desired sort of n-connexity?

As in the former case, I shall answer this question by actually

constructing such a relation. I shall define the relation $R_{\sigma\lambda}$ as the relation such that $R\,\{a_1, a_2, \ldots, a_n\}$ when, and only when,

$$a_1, a_2, \ldots, a_n \in C'R,$$

and the conclusion of (7) is false*.

I shall define the class, ϖ_R, as follows:

(8) $\varpi_R = \hat{\alpha}\,\{x, y \in \alpha \,.\, a_1, a_2, \ldots, a_{n-2} \in C'R \,.\, \supset_{x, y, a_1, a_2, \ldots, a_{n-2}} \,.$
$R_{\sigma\lambda}\{x, y, a_1, a_2, \ldots, a_{n-2}\} \,.\, R_{\sigma\lambda}\{x, a_1, y, a_2, \ldots, a_{n-2}\} \,.\, \ldots$
$R_{\sigma\lambda}\{x, a_1, a_2, \ldots, a_{n-2}, y\} \,.\, R_{\sigma\lambda}\{y, x, a_1, a_2, \ldots, a_{n-2}\} \,.$
$R_{\sigma\lambda}\{a_1, x, y, a_2, \ldots, a_{n-2}\} \,.\, \ldots\, R_{\sigma\lambda}\{a_1, x, a_2, \ldots, a_{n-2}, y\} \,.\, \ldots$
$R_{\sigma\lambda}\{y, a_1, a_2, \ldots, a_{n-2}, x\} \,.\, \ldots\, R_{\sigma\lambda}\{a_1, a_2, \ldots, a_{n-2}, x, y\} ::$
$c \in \alpha \,.\, \supset_c : R_{\sigma\lambda}\{c, b_1, b_2, \ldots, b_{n-1}\} \,.\, \vee \,.\, R_{\sigma\lambda}\{b_1, c, b_2, \ldots, b_{n-1}\} \,.\, \vee \ldots$
$\vee \,.\, R_{\sigma\lambda}\{b_1, b_2, \ldots, b_{n-1}, c\} :. \supset_{b_1, b_2, \ldots, b_{n-1}} \,.\, b_1, b_2, \ldots, b_{n-1} \in \alpha\}$ Df.

Next, I define ins as follows:

(9) $\text{ins} = \hat{P}\hat{Q}\,\{P\,\{\alpha_1, \alpha_2, \ldots, \alpha_n\} \,.\equiv_{\alpha_1, \alpha_2, \ldots, \alpha_n} : \alpha_1, \alpha_2, \ldots, \alpha_n \in \varpi_Q :$
$(\exists a_1, a_2, \ldots, a_n) \,.\, a_1 \in \alpha_1 \,.\, a_2 \in \alpha_2 \ldots a_n \in \alpha_n \,.\, Q\,\{a_1, a_2, \ldots, a_n\}\}$ Df.

Now, I claim, ins'R possesses the desired sort of n-connexity, whatever R may be.

For did it not, by (7), it would be possible to find n distinct α's, say $\alpha_1, \alpha_2, \ldots, \alpha_n$, such that none of those relations hold between them which can be made from those in the conclusion of (7) by substituting ins'R for R, and each α for the a with the same number; while, as we learn from (9), each α is a member of ϖ_R. That is to say, if we pick out one member from α_1, say x_1, one from α_2, say x_2, and so on till we come to α_n, from which we pick out x_n, then $x_1, x_2, \ldots, x_n$ will stand to one another in none of the relations mentioned in the conclusion of (7), and hence will stand to one another in the relation $R_{\sigma\lambda}$. This will be true whatever the values that x_1 takes in α_1, x_2 in α_2, etc. It is easy to see that from this and the second half of the proposition in the brackets in (8), we can conclude that $\alpha_1 = \alpha_2 = \ldots = \alpha_n$, which contradicts our hypothesis. Hence, ins'R always possesses the n-connexity expressed in (7).

Another and equally important property possessed by ins'R is that, if (ins'R) $\{\alpha_1, \alpha_2, \ldots, \alpha_n\}$, $\alpha_1, \alpha_2, \ldots, \alpha_n$ *are all distinct.* For suppose that (ins'R) $\{\alpha_1, \alpha_2, \ldots, \alpha_1, \ldots, \alpha_n\}$. Then we shall have to have, by the definition of ins, $R\,\{a_1, a_2, \ldots, b, \ldots, a_n\}$, where a_1 belongs to α_1, a_2 to α_2, etc., b to α_1, and so on till we get to a_n, which belongs to α_n; $\alpha_1, \alpha_2, \ldots, \alpha_n$ are all, by the definition of ins, members of ϖ_R. Therefore, by the definition of ϖ_R, we shall

* It will be seen, of course, that $R_{\sigma\lambda}$, ϖ_R, and ins are essentially functions of the particular sort of n-connexity asserted in (7).

have $R_{\sigma\lambda}\{a_1, a_2, \ldots, b, \ldots, a_n\}$. We are thus led into a contradiction. It will be noted that this property too is characteristic of ordinary binary serial relations, of ternary relations such as the 'between' relation, and although in this case not clearly stated, of Vailati's separation-relations.

§ 4. Now two interesting questions arise: first, what hypothesis is necessary concerning the n-adic relation R if ins'R is to have a given sort of n-transitivity? and secondly, is it possible to build a function of R which has any given sort of n-transitivity, any given sort of n-connexity, and is such that if this function holds between $\kappa_1, \kappa_2, \ldots, \kappa_n$, the κ's are all distinct? The first question is exceedingly easy to answer. Let the transitivity in question be that of (1), and the connexity that of (7). Modify (1) in the following manner: if in any of the products that, added, make up T_R, a term, say x, occurs as argument to several R's, replace it in all but one of its occurrences by some term, so that in no two occurrences is it replaced by the same term; multiply the product in which it occurs by all the expressions which can be formed by taking $R_{\sigma\lambda}$ [derived from the connexity expressed in (7)], and giving it as arguments any n (not all necessarily distinct) of the terms which replace x, including x itself; and introduce the terms, other than x itself, which replace x, as apparent variables, in such a manner that their range is the whole left side of (1), and that they are preceded by an $\exists$. If we transform (1) in this way, it is easy to see, though tedious to prove, that we obtain a sufficient condition for ins'R's possessing the sort of n-transitivity indicated in (1) and the sort of n-connectedness indicated in (7).

As to the second question, it is almost self-evident that ins'$[(\text{ins}'R)_T]$ possesses the sort of n-transitivity indicated in (1), the sort of n-connexity indicated in (7), and that if

$$\{\text{ins}'[(\text{ins}'R)_T]\}\ \{\kappa_1, \kappa_2, \ldots, \kappa_n\},$$

and $\kappa_i, \kappa_j, i = j$. The two latter properties follow simply from the fact that this relation is an ins of something; the fact that it has the former quality follows obviously from the following considerations. If Q has any sort of n-connexity, and $Q \subseteq P$, then P, a fortiori, has the same sort of n-connexity, if its field is that of Q; for the hypothesis of (7) (with R changed throughout to Q), remains unchanged, while, if

$$Q\{a_1^{(s)}, a_2^{(s)}, \ldots, a_n^{(s)}\}, \text{ then } P\{a_1^{(s)}, a_2^{(s)}, \ldots, a_n^{(s)}\},$$

so that the conclusion of (7) is true of P if it is true of Q. Therefore, $(\text{ins}'R)_T$ has the desired sort of n-connexity and n-transitivity, though it may be possible for us to have $i \neq j$, $\kappa_i = \kappa_j$, and $(\text{ins}'R)_T\{\kappa_1, \kappa_2, \ldots, \kappa_n\}$. Since $(\text{ins}'R)_T$ is connected in the way determined by (7), $[(\text{ins}'R)_T]_{\sigma\lambda}$ can only hold between $\alpha_1, \alpha_2, \ldots, \alpha_\kappa$

when $\alpha_1 = \alpha_2 = \ldots = \alpha_\kappa$. Therefore, $\varpi_{(\mathrm{ins}'R)_T}$ is made up exclusively of unit-classes. Now, we can write the condition for the n-transitivity of $\mathrm{ins}'[(\mathrm{ins}'R)_T]$ as follows:

$$(10)\quad (\exists\lambda_1, \lambda_2, \ldots, \lambda_k)\, T'_{\mathrm{ins}'[(\mathrm{ins}'R)_T]}\{\kappa_1, \kappa_2, \ldots, \kappa_n, \lambda_1, \lambda_2, \ldots, \lambda_k\} .$$
$$\supset_{\kappa_1, \kappa_2, \ldots, \kappa_n} . \mathrm{ins}'[(\mathrm{ins}'R)_T]\{\kappa_1, \kappa_2, \ldots, \kappa_n\}.$$

The expression in the form $T'_{\mathrm{ins}'[(\mathrm{ins}'R)]}$ is here the sum of products of terms of the form $\mathrm{ins}'[(\mathrm{ins}'R)_T]\{\mu_1, \mu_2, \ldots, \mu_n\}$, where the μ's are to be found among the κ's and λ's, and all the λ's appear somewhere as arguments to $\mathrm{ins}'[(\mathrm{ins}'R)_T]$. Therefore, since

$$C'\mathrm{ins}'[(\mathrm{ins}'R)_T] \subset \varpi_{(\mathrm{ins}'R)_T},$$

all the κ's and λ's are unit classes. Therefore, since

$$\{\mathrm{ins}'[(\mathrm{ins}'R)_T]\}\{\nu_1, \nu_2, \ldots, \nu_n\}$$

holds when and only when $\nu_1 \ldots \nu_n$ are members of $\varpi_{(\mathrm{ins}'R)_T}$, and there is an α_1 belonging to ν_1, an α_2 belonging to ν_2, ..., an α_n belonging to ν_n, we may write (10) as follows:

$$(11)\quad (\exists\beta_1, \beta_2, \ldots, \beta_k) . \beta_1, \beta_2, \beta_k, \alpha_1, \alpha_2, \ldots, \alpha_n \in \breve{\iota}''\varpi_{(\mathrm{ins}'R)_T} .$$
$$T_{(\mathrm{ins}'R)_T}\{\alpha_1, \alpha_2, \ldots, \alpha_n, \beta_1, \beta_2, \ldots, \beta_k\} . \supset_{\alpha_1, \alpha_2, \ldots, \alpha_n} . (\mathrm{ins}'R)_T\{\alpha_1, \alpha_2, \ldots, \alpha_n\}.$$

From (5) it follows that (11) is identically satisfied, and hence that $\mathrm{ins}'[(\mathrm{ins}'R)_T]$ has the desired sorts of n-connexity and n-transitivity, and never, to put it roughly, relates a member of its field to itself, *whatever R may be*. Hence, if we have a system whose postulates can be put in the form of three propositions, one asserting a certain n-transitivity, another a certain n-connexity of a given n-adic relation, P, and the third asserting that P never relates a member of its field to itself, then, given any n-adic relation, R, we can construct a function of R having the desired properties of P. Moreover, it is easy to see that if R itself has the desired properties, the constructed relation will be, so we may put it, of the same formal properties as R, but two types higher.

§ 5. Now, there are very important sorts of relations whose definitions may be put in the above form. The general 'between' relation between members of a series is, it is easy to see, *completely* determined as to its formal properties by the three propositions

$$(\exists d) : abd . bdc . \vee . abd . bcd . \vee . adc . dbc .$$
$$\vee . abd . acd . bac . \vee . dab . dac . bac . \vee . bca : \supset_{a,b,c} . abc,$$
$$(\exists m, n) : amn . \vee . man . \vee . mna : bmn . \vee . mbn . \vee . mnb :$$
$$cmn . \vee . mcn . \vee . mnc : \supset_{a,b,c} :$$
$$a = b . \vee . b = c . \vee . c = a . \vee . abc . \vee . bca . \vee . cab,$$
$$abc . \supset_{a,b,c} . a \neq b . b \neq c . a \neq c.$$

Similarly, if we understand the separation-relation to hold only between four distinct terms, the general separation-relation is *completely* determined by the three following propositions:

$$(\exists e) : ab \,||\, dc \,.\, \mathbf{v} \,.\, cd \,||\, ab \,.\, \mathbf{v} \,.\, ab \,||\, ec \,.\, ae \,||\, cd : \supset_{a,b,c,d} \,.\, ab \,||\, cd,$$

$$(\exists m, n, o) : am \,||\, no \,.\, \mathbf{v} \,.\, ma \,||\, no \,.\, \mathbf{v} \,.\, mn \,||\, ao \,.\, \mathbf{v} \,.\, mn \,||\, oa :$$
$$bm \,||\, no \,.\, \mathbf{v} \,.\, mb \,||\, no \,.\, \mathbf{v} \,.\, mn \,||\, bo \,.\, \mathbf{v} \,.\, mn \,||\, ob :$$
$$cm \,||\, no \,.\, \mathbf{v} \,.\, mc \,||\, no \,.\, \mathbf{v} \,.\, mn \,||\, co \,.\, \mathbf{v} \,.\, mn \,||\, oc :$$
$$dm \,||\, no \,.\, \mathbf{v} \,.\, md \,||\, no \,.\, \mathbf{v} \,.\, mn \,||\, do \,.\, \mathbf{v} \,.\, mn \,||\, od :$$
$$\supset_{a,b,c,d} : a = b \,.\, \mathbf{v} \,.\, b = c \,.\, \mathbf{v} \,.\, c = d \,.\, \mathbf{v} \,.\, d = a \,.\, \mathbf{v} \,.\, a = c \,.\, \mathbf{v} \,.\, b = d \,.\, \mathbf{v} \,.$$
$$ab \,||\, cd \,.\, \mathbf{v} \,.\, ac \,||\, bd \,.\, \mathbf{v} \,.\, ad \,||\, bc,$$

$$ab \,||\, cd \,.\, \supset_{a,b,c,d} \,.\, a \neq b \,.\, a \neq c \,.\, a \neq d \,.\, b \neq c \,.\, c \neq d \,.\, b \neq d.$$

That is, *from any triadic or tetradic relation, we are able to construct a between-relation or a separation-relation, respectively.* This fact should play much the same part in explaining how the regular relations of space may be derived from the irregular relations to be found in our experience that the analogous fact concerning dyadic relations plays in showing how the serial relation of the instants of time may be derived from the non-serial relation of complete succession between events*. Logically too this fact has a considerable interest, for it gives a hint of another method of defining mathematical systems than by the use of postulates; given our fundamental logical postulates to start with, we may be able to select the fundamental 'indefinables' of a mathematical system in such a manner that whatever values they may assume within their range of significance, the fundamental formal properties of the system will remain invariant.

§ 6. Of course, *all* the formal properties of a triadic or tetradic relation are not determined when the relation is completely determined as a between or separation relation. Hence there remain interesting and important questions yet as to whether simple properties of R may be given which will give $\mathrm{ins}`R$ or $R_T$ or $\mathrm{ins}`[(\mathrm{ins}`R)_T]$ properties analogous to density or 'Dedekindianness,' etc. If density with respect to a given transitivity, say that of (1), be the property of a relation R which holds when the implication in (1) is converted, then it requires little proof to see that if the converse of (1), modified in the manner that (1) is modified in the first paragraph of § 4, is true of R, and if $C`R \subset s`\varpi_R$, then $\mathrm{ins}`R$ will have the required sort of density. I know of no simpler property of $R$, however, by which we can replace $C`R \subset s`\varpi_R$, and, at any rate, if R is a between or separation relation, this sort of density will not be the property which we would naturally call by that name. If $R\{a, b, c\}$ means 'b is between a and c,'

* See *Proc. Camb. Phil. Soc.*, vol. XVII, Part 5, pp. 441—9.

then what we would naturally call density would be the property of R which can be written

$$(a, c) :: a, c \in C`R \,.\, \supset :. (\exists b) \,.\, R`\{a, b, c\} : \vee \,.\, a = c.$$

Provided that $C`P \subset s`\varpi_P$, then if P is any triadic relation having this property, then $\text{ins}`P$, and hence, as may be seen easily, $(\text{ins}`P)_T$ and $\text{ins}`[(\text{ins}`P)_T]$, will have this property.

§ 7. Let us now turn to the second topic to be treated in this paper, the problem of the synthesis of the series of sensation-intensities from the relations between sensations given in experience. This problem, in itself, is not one of pure logic or of pure mathematics, but its solution depends upon the solution of a purely logical and mathematical problem. In my previous article*, as I said at the beginning of this paper, I showed how from the relation of complete succession between the events in time, we can construct the series of the instants in time. The method was the following: we make the definitions:

$$(12) \qquad P_{se} = (\dot{-}P \dot{-} \breve{P}) \restriction C`P \qquad \text{Df.}$$

$$(13) \qquad \tau_P = \hat{\alpha}\{\alpha = p`\overrightarrow{P}_{se}``\alpha\} \qquad \text{Df.}$$

$$(14) \qquad \text{inst} = \hat{Q}\hat{P}\{Q = (\breve{\epsilon}; P) \restriction \tau_P\} \qquad \text{Df.}$$

If P is the relation between two events, x and y, when x is over before y begins, then P_{se} is the relation between two events which occur together at some moment; τ_P is the class of all instants of time—that is, the class of all those classes, α, such that α is made up of events in such a manner that every two events in α occur together at some moment, and if an event occurs at the same moment with every member of α, then it belongs to α; and $\text{inst}`P$ is the relation between two members of τ_P—that is, instants—when some event at the first instant is over before some event at the second instant begins: that is, it is the relation between an instant and a succeeding instant. If $P \mid P_{se} \mid P \subseteq P$, whether P is a temporal relation or not, $\text{inst}`P$ will be a series. Now, let P stand for the relation, say, between any coloured object and a noticeably brighter one. Then P_{se} will be the relation between two coloured objects when the first is apparently of the same brightness as the second, for it is the relation between two members of the field of P—that is, coloured objects—when neither is in the relation P to the other. Now, it is obvious that when $xP \mid P_{se}y$, x must be, noticeably or unnoticeably, more bright than y, for this proposition says that x is noticeably brighter than some object which, at the brightest, is indistinguishable from y. Therefore, it is obvious that if $xP \; P_{se} \mid Py$, x is brighter than something noticeably brighter than y, and hence is noticeably brighter than

* See *Proc. Camb. Phil. Soc.*, vol. XVII, Part 5, pp. 441—9.

y, and $P \mid P_{se}\ P \subseteq P$. inst'$P$ is therefore here also a series, and nothing would seem more natural than for us to call it the series of sensation intensities.

But there are serious objections against this method of procedure, and here a genuine logical problem arises. For, although it is natural to regard a sensation-intensity as a class of sense-objects—the class of sensations 'of a certain intensity'—we naturally consider the intensity of a given sensation as uniquely determined, and the relations between two sensations, x and y, when x is of the same intensity as y, as a transitive, symmetrical, reflexive relation. Now, in general, τ_P is not a class of mutually exclusive classes, and the relation between two terms which belong to the same member of τ_P is not transitive. The fact that a certain river was flowing during the Siege of Troy, and is flowing while I am writing this article, does not mean that I was writing this article during the Siege of Troy, yet if we take P as the relation between one event and another which completely follows it, my writing this article and the flowing of the river will both belong to some member of τ_P; the Siege of Troy and the flowing of the river will both belong to some other member of τ_P. So we have the definite mathematical problem before us: given a relation, P, fulfilling certain conditions, not sufficient to make it a series, we wish to construct from it a serial relation in such a manner that the terms of this series shall form a class of mutually exclusive classes.

I shall first give the method by which this series may be derived from the relation between x and y when x is of noticeably greater intensity than y; then I shall state a set of conditions sufficient to secure the serial character of the derived relation, and finally I shall interpret conditions and results. Perhaps the best method logically would be first to formulate all the conditions to which the original relation must be subject, and then to treat the problem as a purely formal one, but the logical gain would hardly compensate us for the loss in clarity. So I first make the following definitions:

$$(15) \qquad P_s = (\overrightarrow{P_{se}} \mid \overrightarrow{P_{se}}) \upharpoonright C'P \qquad \text{Df.}$$

$$(16) \qquad \lambda_P = D'\overrightarrow{P_s} \qquad \text{Df.}$$

$$(17) \qquad \text{int} = \hat{Q}\hat{P}\{Q = [\breve{\epsilon}; (P_{se} \mid P)] \upharpoonright \lambda_P\} \qquad \text{Df.}$$

If P is the relation between x and y when x is, say, noticeably brighter than y, then P_{se} is the relation between two things which are not distinguishable as concerns their brightness, and P_s is the relation between two things possessing brightness when each of the things which is indistinguishable from the one in brightness is also indistinguishable from the other, and *vice versa*.

It follows at once from the definition of P_s that it is transitive, symmetrical, and reflexive, whatever P may be, and hence in this respect it satisfies the requirements we have set up for the relation between two members of a sensation-intensity.

λ_P is the class of brightness-intensities, where P is the relation 'noticeably brighter than.' Since λ is defined as $\mathrm{D}`\overrightarrow{P_s}$, it follows that it must always be a class of mutually exclusive classes; for suppose that two members of $\lambda_P$, say $\overrightarrow{P_s}`x$ and $\overrightarrow{P_s}`y$, had the term $z$ in common. Then we would have $zP_sx$ and $zP_sy$. From the definition of $P_s$ it is symmetrical, so we get $xP_sz$ and $zP_sy$, which, on account of the transitivity of $P_s$, gives us $xP_sy$, and, hence, $\overrightarrow{P_s}`x \subset \overrightarrow{P_s}`y$. In just the same way, we get $\overrightarrow{P_s}`y \subset \overrightarrow{P_s}`x$, or, finally, $\overrightarrow{P_s}`y = \overrightarrow{P_s}`x$. $\mathrm{int}`P$ is the relation between two members of λ_P when a member of one is in the relation $P_{se}\ P$ with a member of the other. Whatever P is, $\mathrm{int}`P \subseteq J$. For suppose that $\alpha\,(\mathrm{int}`P)\,\alpha$. Then, since α must belong to λ_P, every term of α stands in the relation P_s to every term of α. However, from the definition of $\mathrm{int}`P$, there must be two terms of α, x and y, such that $xP_{se}\,|\,Py$. This may be written as

$$(\exists z)\,.\,xP_{se}z\,.\,zPy.$$

From this and the definition of P_{se}, we get $(\exists z)\,.\,xP_{se}z\,.\,z \sim P_{se}y$, or $\overrightarrow{P_{se}}`x \neq \overrightarrow{P_{se}}`y$, which may be written $x \doteq P_sy$. Thus, the assumption that $\sim(\mathrm{int}`P \subseteq J)$ is self-contradictory.

A condition which will ensure the transitivity of $\mathrm{int}`P$ is $P_{se}\,|\,P \in \mathrm{trans}$. For it follows from the definitions of $P_s$, $\lambda_P$, and int that if $\alpha\,(\mathrm{int}`P)^2\,\beta$, $\alpha\,[\{\breve{\epsilon}\,;(P_{se}\,|\,P\,|\,\overrightarrow{P_{se}}\ \overrightarrow{P_{se}}\ P_{se}\,|\,P)\} \upharpoonright \lambda_P]\beta$.

Now,

$$(18)\quad \vdash .\ \breve{\overrightarrow{P_{se}}}\,;\overrightarrow{P_{se}}\ P_{se} = \hat{x}\hat{z}\,\{(\exists\alpha, y)\,.\,\alpha = \overrightarrow{P_{se}}`y\,.\,\alpha = \overrightarrow{P_{se}}`x\,.\,yP_{se}z\}$$
$$= \hat{x}\hat{z}\,\{(\exists y)\,.\,\overrightarrow{P_{se}}`y = \overrightarrow{P_{se}}`x\,.\,z \in \overrightarrow{P_{se}}`y\} = P_{se}.$$

Therefore, $P_{se}\,|\,P\ \breve{\overrightarrow{P_{se}}}\,|\,\overrightarrow{P_{se}}\,|\,P_{se}\,|\,P$ is simply $P_{se}\ P\,|\,P_{se}\ P$. If $P_{se}\,|\,P$ is transitive, then we find that $\alpha\,(\mathrm{int}`P)^2\,\beta$ implies that $\alpha\,[\{\breve{\epsilon}\,;(P_{se}\,|\,P)\} \upharpoonright \lambda_P]\,\beta$, which is simply $\alpha\,(\mathrm{int}`P)\beta$. A hypothesis which will make $P_{se}\,P$ transitive is $P\,|\,P_{se}\,|\,P \subseteq P$. This is the same condition which we found to suffice for the transitivity of $\mathrm{inst}`P$.

When will $\mathrm{int}`P$ be connected? Under what conditions, that is, will it be true that

$$\alpha, \beta \in C`\mathrm{int}`P\,.\,\alpha \neq \beta\,.\,\supset_{\alpha,\beta} : \alpha\,(\mathrm{int}`P)\,\beta\,.\,\vee\,.\,\beta\,(\mathrm{int}`P)\,\alpha\,?$$

This condition is manifestly implied by

$$\alpha, \beta \in \lambda_P \,.\, \alpha \neq \beta \,.\, \supset_{\alpha,\beta} : \alpha\,(\mathrm{int}`P)\,\beta \,.\, \vee \,.\, \beta\,(\mathrm{int}`P)\,\alpha.$$

Since $\alpha\,(\mathrm{int}`P)\,\beta$ merely demands that $\alpha$ and $\beta$ should be members of $\lambda_P$, and that *some* member of $\alpha$ should bear the relation $P_{se}\ P$ to *some* member of $\beta$, and since if $x$ and $y$ are both members of $\alpha$, and $\alpha \in \lambda_P$, $xP_sy$, $\mathrm{int}`P$ will be connected if

$$x \doteq P_s y \,.\, \supset_{x,y} : xP_{se} \mid Py \,.\, \vee \,.\, yP_{se} \mid Px.$$

Now,

$$(19)\quad \vdash :: x \doteq P_s y :. \supset_{x,y} :. \overrightarrow{P}_{se}`x \neq \overrightarrow{P}_{se}`y :.$$
$$\supset_{x,y} :. (\exists z) : zP_{se}x \,.\, z \doteq P_{se}y \,.\, \vee \,.\, zP_{se}y \,.\, z \doteq P_{se}x :.$$
$$\supset_{x,y} :. (\exists z) : zP_{se}x \,.\, zPy \,.\, \vee \,.\, zP_{se}x \,.\, yPz \,.\, \vee \,.\, zP_{se}y \,.\, zPx \,.\, \vee \,.\, zP_{se}y \,.\, xPz :.$$
$$\supset_{x,y} :. xP_{se} \mid Py \,.\, \vee \,.\, yP \mid P_{se}x \,.\, \vee \,.\, yP_{se} \mid Px \,.\, \vee \,.\, xP \mid P_{se}y.$$

If $P \mid P_{se} \subseteq P_{se} \mid P$, this reduces at once to the condition that we have just shown to be sufficient for the connectedness of $\mathrm{int}`P$.

§ 8. We have seen, then, that if

$$P_{se} \mid P \in \mathrm{trans} \quad \text{and} \quad P \mid P_{se} \subseteq P_{se} \mid P, \quad \mathrm{int}`P \in \mathrm{ser}.$$

Now the questions arise, what do these conditions mean when P is, for example, the relation 'noticeably brighter than'? and, are they true of such relations? The meaning of $P_{se} \mid P \in \mathrm{trans}$ in such a case is clear, as is also its truth; $P_{se} \mid P$ is the relation between two objects, x and y, when x is not merely apparently, but actually brighter than y, for $xP_{se}\ Py$ says that x is only subliminally different, if at all different, in brightness from something that is supraliminally brighter than y. Now, the transitivity of the relation, 'brighter than,' is obvious: at least as obvious, at any rate, as the existence of a series of brightnesses.

The meaning of $P \mid P_{se} \subseteq P_{se} \mid P$, however, is not quite so obvious. This condition demands that if x be noticeably brighter than something indistinguishable from y, it shall be indistinguishable from something noticeably brighter than y. We may interpret this demand as saying: if x is noticeably brighter than everything noticeably less bright than y, then y is noticeably less bright than everything noticeably brighter than x. A little reflection will convince us that this proposition is probably true: moreover, it is easy to see that its truth, and the truth of analogous propositions concerning all sorts of sensory intensity, form necessary conditions for the truth of the Weber-Fechner law. For suppose that this proposition were false: we might then have, to put it crudely, x and y both just noticeably brighter than x, and u just noticeably brighter than x, but subliminally different from y. Let a be the objective strength of the stimulus produced by z; then, by Weber's law, the strength of the stimulus produced by x or y will be $a\,(1 + c)$, where c is a constant

independent of the value of a. Since u is just noticeably brighter than x, the strength of stimulus produced by u will be

$$a(1+c)(1+c) = a(1+2c+c^2).$$

But since u is only subliminally different from y in brightness, the strength of the stimulus produced by u is less than $a(1+2c+c^2)$.

Hence, we are landed in the contradiction,

$$a(1+2c+c^2) < a(1+2c+c^2).$$

A little reflection will convince the reader that any other way of violating the condition, $P \mid P_{se} \subseteq P_{se} \mid P$, would likewise be incompatible with Weber's law.

This seems the proper place to call attention to the fact that if P be the relation of complete precedence between the events in time, $P \mid P_{se} \subseteq P_{se} \mid P$ is *false*. For suppose that at this present moment two events begin, one of which lasts five minutes and the other ten. It is clear that neither event can be simultaneous with an event which wholly precedes the other: that is, neither bears to the other the relation $P_{se} \mid P$. Now suppose that one minute after the shorter event is ended, some event begins. This bears the relation P_{se} to the longer event, and the shorter event bears to it the relation P. Therefore, the shorter event bears to the longer event the relation $P \mid P_{se} \doteq P_{se} \mid P$. So we have proved nothing in this article which entitles us to say that if P is the relation of complete precedence among the instants of time, int'P is a series. And, as a matter of fact, it is not a series. If, however, we limit the field of P to events, say, that last exactly five minutes, then $P \mid P_{se} \subseteq P_{se} \mid P$, and int'$P$ is a series.

In case P is the relation, 'noticeably brighter than,' one can readily see that int'P is not only a series, but the series we mean when we speak of the series of brightnesses. For, if Weber's law is true, or even if some quantitatively different law of the same general form is true, P_s is exactly the relation which holds between two things of the same brightness, for xP_sy says, practically, the limina of distinguishability from x are the limina of distinguishability from y, and it can be deduced from this and Weber's law that this is true when and only when x and y produce stimuli of the same intensity, and hence it follows further from Weber's law, x and y must be of the same sensation-intensity. λ_P is therefore the class of all classes containing all the things of the same brightness as a given thing, and hence can be fittingly called the class of all brightnesses; and what could be more natural than to say that a given brightness is greater than another when and only when a thing of the first brightness is brighter than a thing of the second?

If we want to secure the compactness of int'P, it is sufficient to assume the compactness of $P_{se} \mid P$, though not, as far as I know, necessary. Similarly, $P_{se} \, P \, \epsilon \, \mathrm{Ded}$ is a condition sufficient to assure the Dedekindian character of int'P.

The interest and importance of this work on sensation-intensities lies in the fact that it is often naively assumed by psychologists that the series of sensation-intensities is in some wise a datum of experience, and not a construction. As a result, they are led into the most grotesque interpretations of such numerical formulae as Weber's law. A series of sensation-intensities is often treated as if it were, in some sense or other, a series of sensation-*quantities*, without any analysis whatsoever of the basis on which this series is put into one-one correspondence with the series of 0 and the positive real numbers, in order of magnitude. It is at any rate a necessary preliminary to this exceedingly complex problem to know what the series of sensation-intensities really are, and what their relation to our experience is: without this analysis, no scientific psychophysics is possible.

The Determination of the Prime or Composite Nature of Large Numbers by Fermat's Theorem. By H. C. POCKLINGTON, M.A., St John's College.

[*Read* 9 March 1914.]

1. Probably the best way to find out whether a large number N is composite is to take some number x and find the least residue of x^{N-1} to modulus N. Of course we first raise x to powers the indices of which are powers of 2 by repeated squaring and division by the modulus to find the remainder, and then multiply numbers chosen from the results so that their indices add up to $N-1$. If x^{N-1} is not $\equiv 1$ mod. N we infer from Fermat's Theorem that N is composite. The object of this paper is to explain how to proceed if we find that $x^{N-1} \equiv 1$ mod. N.

2. Let p be a prime divisor (preferably the largest) of $N-1$, contained a times in it, and let $(N-1)/p = m$. We now find the remainder of x^m to divisor N by the method already given. If the remainder is not unity we subtract 1 from it and find the greatest common divisor δ of the result and N. If $\delta \neq 1$ we have of course found a factor δ of N. If $\delta = 1$ we see that $x^m - 1$ is not divisible by any prime divisor of N. Hence the exponent to which x belongs with respect to any prime divisor π of N, which we have found to divide $N-1$, does not divide $(N-1)/p$. Hence it contains p^a as a factor. But $\pi - 1$ is divisible by this exponent. Hence $\pi - 1$ is divisible by p^a, so that all the prime divisors of N are of the form $kp^a + 1$. If now p^a is greater than $\sqrt{N}$ this shows that N has no divisor less than its square root, so that N must be prime. If p^a is not too many times smaller than $\sqrt{N}$ we shall only have to try a comparatively small number of possible divisors. If p^a is too small we proceed in the same way with another prime factor q of N, and will probably succeed in finding a factor of N or in showing that every prime divisor of N is of the form $kp^a q^b + 1$.

It may happen* that $x^m \equiv 1$ mod. N. In this case we take any prime factor q of m (preferably the largest) and proceed just as before with m replaced by m/q. It can happen that x belongs to a small exponent with respect to N, for example 2 belongs to exponent p, modulus $2^p - 1$. In such a case we can only take a new value of x. We can of course, after proving that N is prime, continue the process and find the exponent to which x belongs with respect to N.

* This can only happen rarely, for if N is prime the number of distinct values of x for which this happens is only $1/p$ of their number.

3. The method can be applied with exceptional ease to Fermat's numbers $F = 2^m + 1$, where $m = 2^\lambda$. Here 2 is always an unsuitable number. We may choose $x = 3$. Then, if in the course of the work we find a remainder equal to unity, we know that F must be composite, for 3 is a primitive root of F whenever F is prime. Also if the mth squaring does not give remainder unity it is clear that F is composite. If however the mth squaring is the first that gives remainder unity, then F is prime. For our work shows that the exponents to which 3 belongs with respect to the prime divisors of F all divide 2^m, and that one at least does not divide 2^{m-1}. Hence one exponent is 2^m and the corresponding prime is at least $2^m + 1 = F$.

The method can also be applied very easily to Mersenne's numbers $M = 2^p - 1$, where p is prime. Here also 2 is always unsuitable. If M is prime we must have

$$x^{(M+1)/2} \equiv \left(\frac{x}{M}\right) x \text{ mod. } M,$$

where $\left(\frac{x}{M}\right)$ is Jacobi's symbol. We see *e.g.*, that if we wish to test $2^{89} - 1$ (the primeness of which is in dispute), we shall have to make 88 multiplications and about the same number of divisions, the multipliers, multiplicands and the divisor having about 27 digits, and if the number is prime we shall have to do a further amount of work not easy to estimate but not likely to greatly exceed that already done.

4. This method has the disadvantage that we only (excepting in rare cases) determine whether N is prime or composite, and that we may require to factorize $N - 1$ in part at least. The advantage lies in the fact that the labour increases approximately as $(\log N)^3$, not as $\sqrt{N}$, which makes it a much easier method than that of the Idoneals if N is large. It is also well adapted for use with the arithmometer. Other methods of proving that a number is prime only give negative evidence of the fact (absence of any divisor or absence of a second representation in a quadratic form), so that a single slip can cause a composite number to be taken as prime. In the present method however it is hardly possible that we should find $x^{N-1} \equiv 1$ mod. N by making a slip, and the accuracy of such a result as $x^{(N-1)/p} \equiv u$ mod. N can be checked by finding what u^p is congruent to.

PROCEEDINGS

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*Experiment on the harmonic motion of a rigid body**. By G. F. C. SEARLE, Sc.D., F.R.S., University Lecturer in Experimental Physics, Fellow of Peterhouse.

[*Read* 24 November 1913.]

1. *Introduction.* One of the problems which confront a demonstrator of experimental physics is to teach the principles of harmonic motion in such a way that they shall be absorbed by the students and shall be incorporated in their mental equipment. For some reason, not easy to understand, harmonic motion does present real difficulties to students, but every effort should be made to clear away these difficulties, since many important methods of determining physical quantities depend upon measurements made on bodies which vibrate harmonically. The experiment described below is designed to illustrate the principles of harmonic motion in the case of a rigid body vibrating about a vertical axis under the influence of the torsion of a wire.

2. *Theory of experiment.* Let a rigid body be suspended from a fixed support by a vertical torsion wire and let the moment of inertia of the body about the axis of the wire be M gm. cm.^{2}. Let the couple required to turn the rigid body through one radian against the torsion of the wire be μ dyne-centimetres. Then, when the angle is θ radians, the couple is $\mu\theta$ dyne-cm., and, when the body is free to move, the angular acceleration of the body

* The description of this experiment has been reproduced in a small manual on *Experimental Harmonic Motion* which is now (Nov. 2, 1914) in the press. The manual will be published by the Cambridge University Press and is similar in character to the author's *Experimental Elasticity*.

is $\mu\theta/K$ radians per sec. per sec. towards the equilibrium position. The motion is therefore harmonic and the periodic time is given by

$$T = 2\pi\sqrt{\frac{K}{\mu}} \text{ seconds} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1).$$

In the experiment, K is found by weighing and measuring the rigid body and μ is found from a series of measurements of the angle through which the lower end of the wire is turned by a series of couples applied statically. The periodic time is calculated by (1) and this time is compared with that which is observed when the body is allowed to vibrate. The agreement between the observed and the calculated values of the periodic time forms an experimental test of the accuracy of the dynamical principles employed in the calculation.

3. *The vibrating system.* It is essential that the torsion wire should be properly secured (1) to the fixed support, and (2) to the vibrating body. This result is best obtained by soldering each end of the wire into a hole drilled along the axis of a cylindrical rod a few centimetres in length and about 0·5 cm. in diameter.

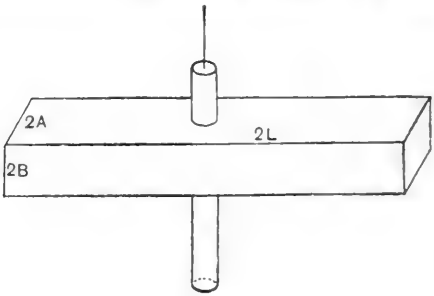

Fig. 1.

One of these rods is secured by a set screw to the fixed support and the other is secured by a set screw in a hole drilled in any body which is to be suspended by the wire. These rods are so much stiffer than the torsion wire that small variations in the positions of the points at which the set screws press upon them make little difference in the couple required to turn the suspended body through one radian against the torsion of the wire. Care should, however, be taken that the set screws, which fix (1) the vibrating system and (2) the cylinder shown in Fig. 2, make contact with the rod at nearly the same point. The torsion wires used in the author's practical class at the Cavendish Laboratory are of steel and are about 32 cm. in length, and 0·175 cm. in diameter.

A convenient rigid body is a rectangular bar (Fig. 1) of length

$2L$ cm., of width $2A$ cm., and of depth $2B$ cm. A hole, into which the cylindrical rods attached to the torsion wire fit, is drilled through the bar at right angles to the plane of the edges $2L$, $2A$, as in Fig. 1. The effective mass of the bar, M grammes, should be marked upon the bar. This is the mass of the bar *before* the hole was drilled through it or the set screw was fitted to it*.

The moment of inertia of the bar is calculated by the formula

$$K = \tfrac{1}{3}M(L^2 + A^2) \text{ gm. cm.}^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2).$$

The mass of the metal taken out of the hole and the mass of the set screw are appreciable in comparison with the mass of the bar itself, and hence, if the actual mass of the bar after it has been drilled and fitted with a set screw were used in formula (2) an appreciable error in K would result. But the hole and the set screw are so close to the axis of vibration that the *moments of inertia* of the metal taken from the hole and of the set screw about the axis are quite inappreciable compared with that of the bar itself, and thus the moment of inertia of the bar as actually used does not differ appreciably from that given by (2), provided that by M is understood the mass of the bar *before* the hole was drilled through it or the set screw was fitted to it.

The moment of inertia of the cylindrical rod soldered to the wire is quite negligible in comparison with that of the bar.

One of the heavy compound laboratory stands supplied by W. G. Pye and Co., of Cambridge, forms a convenient support for the upper end of the torsion wire. Whatever support is used should be rigid and free from shake.

The periodic time of the inertia bar should be deduced from two or three observations of the time occupied by at least 100 complete vibrations, and the time-piece should, if necessary, be compared with a reliable clock.

4. *Determination of the relation between couple and angle.* In the determination of the couple required to twist the lower end of the wire through one radian, the inertia bar is removed from the torsion wire and a cylinder is substituted, as shown in Fig. 2. The couple is applied by means of a thread passing over two ball-bearing pulleys and supporting two small scale pans; it is convenient to adjust the mass of each pan to be 10 gm. A loop is made in the thread and this loop is passed over the set screw securing the cylinder to the rod at the end of the torsion wire.

Care must be taken that the parts of the thread between the cylinder and the pulleys are parallel and horizontal so that, when the two loads are equal, the threads may exert a pure couple on

* Searle, *Experimental Elasticity*, Note VII.

the cylinder. Special care must be taken to ensure that the threads are *tangential* to the cylinder.

If the diameter of the cylinder be D cm. and that of the thread be d cm., the distance between the axes of the two threads is

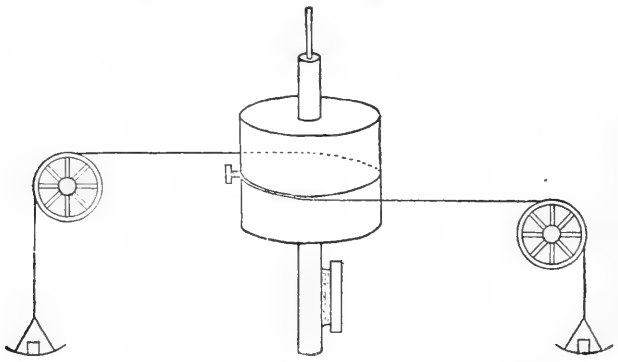

Fig. 2.

$D + d$ cm. and hence, when the load on each thread is m gm., the couple is

$$G = mg\,(D + d) \text{ dyne-cm.} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3).$$

The angle θ, through which the couple G causes the cylinder to revolve, is determined by aid of the simple goniometer shown in Fig. 3.

This instrument was designed in conjunction with W. G. Pye and Co. to provide a means of measuring angles up to about $\frac{1}{4}$ radian (about 14°) with an accuracy of $\frac{1}{4000}$ radian.

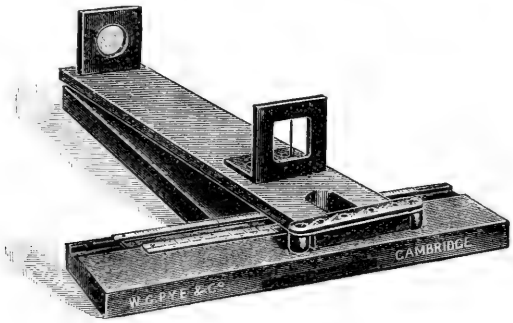

Fig. 3.

The base is formed of a strip of wood furnished at one end with a spherical pivot and at the other with a cross-bar carrying a scale. Angles are measured by means of a movable arm which turns at one end about the pivot, while the other end moves over the scale on

the cross-bar. The optical system consists of a lens fixed to the arm near the pivot and of a fine vertical wire attached to the other end of the arm and adjusted to be in the focal plane of the lens.

The spherical pivot is a phosphor-bronze ball attached to the base by a fitting which allows the distance between the ball and the scale to be adjusted. The ball enters a conical hole turned out of a block of brass attached to the arm. This arrangement destroys three out of the six degrees of freedom of the arm relative to the base. The other end of the arm carries two brass feet which rest upon the cross-bar and thus destroy two degrees of freedom. The remaining degree of freedom allows the arm to turn about an axis through the centre of the ball and perpendicular to the plane of the surface of the cross-bar. This design has the advantage that it is impossible to strain the instrument by lifting it by the movable arm, for the arm at once comes away from the base.

The scale on the cross-bar is divided into millimetres, and the ball is adjusted so that its centre is 40 cm. from the edge of the scale. The readings are taken by means of a fine wire passing across an opening in the arm and stretched by a spring; the wire is easily replaced if broken. The scale is engine-divided on white metal and is provided with an anti-parallax mirror. For small angles, one centimetre along the scale corresponds to $\frac{1}{40}$ radian; as the scale can be read to $\frac{1}{100}$ cm., the angle can be read to $\frac{1}{4000}$ radian, or to about $\frac{1}{70}$ degree.

The lens attached to the movable arm is achromatic and has a focal length of 35 cm. The vertical wire is held in an adjustable frame attached to the arm and is kept tight by a spring, and this frame is adjusted so that the wire is in the focal plane of the lens. The image of a distant point will then fall upon the wire, if the arm be properly directed. If a plane mirror be placed so that the lens lies between it and the wire, the image of the wire formed by two refractions through the lens and one reflection at the mirror may be made to coincide with the wire itself.

When the goniometer is used in mechanical experiments to determine the angle turned through by a body about a vertical axis, a plane mirror is attached to the body and the image of the goniometer wire is made to coincide with the wire itself. If this coincidence is restored after the body has turned, the angle turned through by the body is equal to that turned through by the goniometer arm. To facilitate the adjustment, the short scale which is provided with the instrument is fitted into the frame holding the wire, the divided face of the scale being turned towards the lens. On looking at the wire in the direction of the lens, an inverted image of the scale (formed by the lens and the plane mirror) will be seen crossing the wire. In this use of the

instrument, all that is necessary is that the mirror should be nearly vertical and that the rays from the wire, after passing through the lens, should fall upon the mirror. No other adjustments are required, and the centre of the spherical pivot need not lie on the vertical axis about which the body turns.

The goniometer is placed so that its lens is three or four centimetres from the mirror carried by the suspended system.

In the present experiment, a plane mirror is attached to the suspended system in the manner shown in Fig. 2, by means of soft wax, or, more conveniently, by means of the simple device shown in Fig. 4, in which the mirror is attached to a horizontal

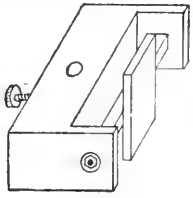

Fig. 4.

axis and is thus capable of easy adjustment. The mirror is adjusted so that it is possible to see the inverted image of the small scale attached to the goniometer arm crossing the vertical wire of the goniometer. The goniometer is securely fixed so as to be free from shake and from liability to accidental displacement. The *base* is adjusted so that when the arm is in its central position, the image of the wire coincides as nearly as possible with the wire itself. The *arm* is then adjusted so that the image of the wire exactly coincides with the wire itself, and the reading of the indicating wire on the *edge* of the scale on the cross-bar of the goniometer is taken.

The thread is then attached to the cylinder and is passed over the pulleys, carrying the pans alone. The goniometer arm is then moved until the goniometer wire again coincides with its own image, and the reading on the scale is taken. The load on each end of the thread is then increased by steps of 10 gm. and the observations are repeated for each load.

It is easily seen that the goniometer arm turns through the same angles as the cylinder, when it is properly adjusted at each stage.

If the reading on the scale of the goniometer for any position of the arm differs from the reading when the arm is in the central position by x cm., and if the distance from the centre of the spherical pivot to the *edge* of the scale be p cm., then the angular displacement of the arm is θ radians, where $\tan\theta = x/p$.

Hence $$\theta = \tan^{-1} x/p.$$

When x/p is small, we have

$$\theta = \frac{x}{p} - \frac{1}{3}\frac{x^3}{p^3} + \frac{1}{5}\frac{x^5}{p^5} - \ldots.$$

If x/p is less than 1/20, it will generally be sufficient to put $\theta = x/p$.

When x/p is greater than 1/20 it will be best to calculate the value of tan θ and from this to find θ in *degrees*. Bottomley's tables may then be used to find the value of θ in *radians*.

On dividing each value of θ by the corresponding value of m, the result will be very nearly constant, thus showing that the angle is proportional to the couple. The mean value of θ/m is then found and is used in the calculation of μ from the formula

$$\mu = \frac{G}{\theta} = \frac{g(D+d)}{\theta/m}.$$

Using this value of μ and the value of K already found from the dimensions of the inertia bar, the time of vibration of the bar is calculated by the formula

$$T = 2\pi\sqrt{\frac{K}{\mu}}.$$

5. *Practical example.* The observations may be entered as in the following record of an experiment by G. F. C. Searle, Oct. 1906.

Load gm.	Reading cm.	x cm.	tan θ	θ degrees	θ radians	$1000\,\theta/m$ radians/gm.
0	15·00	0	0	0	0	0
10	14·43	0·57	0·0142	*	0·0142	1·420
20	13·87	1·13	0·0282	*	0·0282	1·410
30	13·29	1·71	0·0426	*	0·0426	1·420
40	12·71	2·29	0·0571	*	0·0571	1·428
50	12·12	2·88	0·0718	4° 6′	0·0716	1·432
60	11·55	3·45	0·0860	4° 55′	0·0858	1·430
70	10·95	4·05	0·1010	5° 46′	0·1007	1·439
80	10·38	4·62	0·1152	6° 34′	0·1146	1·432
90	9·81	5·19	0·1294	7° 22′	0·1286	1·429
100	9·20	5·80	0·1446	8° 14′	0·1437	1·437

* θ put equal to tan θ here.

Mass of inertia bar $= M = 826$ gm.
Length of inertia bar $= 2L = 37{\cdot}88$ cm.

Width of inertia bar $=2A=1{\cdot}60$ cm.
Moment of inertia of bar $=K=\frac{1}{3}M(L^2+A^2)=9{\cdot}894\times10^4$ gm. cm.^{2}.
Distance from scale to nearer side of pivot $=39{\cdot}92$ cm.
Diameter of pivot $=0{\cdot}38$ cm.
Hence $p=39{\cdot}92+\frac{1}{2}\times0{\cdot}38=40{\cdot}11$ cm.†.
Diameter of cylinder $=D=3{\cdot}15$ cm.
Diameter of thread $=d=0{\cdot}03$ cm.
Mean value of $\theta/m=1{\cdot}428\times10^{-3}$ radians per gm.

Hence, $\mu=\dfrac{g\,(D+d)}{\theta/m}=\dfrac{981\times3{\cdot}18}{1{\cdot}428\times10^{-3}}=2{\cdot}185\times10^6$ dyne-cm. per radian.

Thus, by (1), $T=2\pi\sqrt{\dfrac{K}{\mu}}=2\pi\sqrt{\dfrac{9{\cdot}494\times10^4}{2{\cdot}185\times10^6}}=1{\cdot}338$ sec.

Direct observations with a good stop-watch gave 100 vibrations in 134·2, 133·7, 133·7 seconds. The mean value $T=1{\cdot}339$ agrees very closely with that deduced from the statical experiments.

† In the goniometer used in this experiment p was not capable of adjustment. In the instrument shown in Fig. 3, p is adjusted to be 40·00 cm.

Some Insect Flagellates introduced into Vertebrates. By H. B. FANTHAM, D.Sc., B.A., Christ's College, Cambridge, and Liverpool School of Tropical Medicine, and ANNIE PORTER, D.Sc., Beit Memorial Research Fellow, Quick Laboratory, Cambridge.

(Plate I.)

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Contents.

I. *Introduction.*

For some years past, much attention has been devoted to the protozoal parasites that produce disease in vertebrates and to the means whereby the minute organisms reach their hosts. In the latter connection, the *rôle* of insects in the spread of disease has been investigated and much necessary work has been done in elucidating the life-histories, more particularly of the parasitic Flagellates peculiar to the insects alone, and having no connection with vertebrate maladies. It has been generally considered that such natural flagellates of insects are harmless to their host, and in several cases, notably in the case of *Herpetomonas pediculi* (Fantham), parasitic in the human louse, and *Crithidia pulicis* (Porter), occurring in the human flea, they have been shown to have no deleterious effect when introduced into the human system. However, all flagellates peculiar to insects are not innocuous, should they reach the vertebrate on which their host feeds. By a series of interesting experiments Laveran and Franchini have recently shown that an experimental leishmaniasis (herpetomoniasis) can be induced in mice and rats by inoculating or feeding them with *Herpetomonas pattoni* (Swingle), parasitic in the gut of rat-fleas, and in dogs by inoculating them with *H. ctenocephali* (Fantham), parasitic in dog-fleas, among others. The introduction of the flagellate into the vertebrate produced disease, and death

occurred in some cases. The experiments of these authors tend certainly to support the hypothesis that the flagellates parasitic in the blood of mammals originated in insect hosts, and also show the possibility that other flagellates, equally ranked as harmless to the Metazoa, may acquire pathogenic properties when introduced into strange hosts.

In order to test whether this were so, a series of experiments was undertaken by us during the past year. While Laveran and Franchini often used nearly associated insects and vertebrates, *e.g.* rat-fleas and rat or mouse, dog-fleas and dog, we made a wide divergence, in order to ascertain whether an insect flagellate introduced into a quite unassociated vertebrate might become pathogenic. For this purpose, *Herpetomonas jaculum* (Léger), parasitic in the "water scorpion," *Nepa cinerea*, was chosen as the flagellate, while very young mice served as vertebrate hosts. We also succeeded in infecting a puppy by feeding it on parasitised dog-fleas.

II. *The Life-cycle of* HERPETOMONAS JACULUM *in* NEPA CINEREA.

Herpetomonas jaculum, parasitic in the alimentary tract of *Nepa cinerea*, is a flagellate possessing also a non-flagellate stage in its life-cycle. This non-flagellate form is an ovoid, Leishmania-like resistant body that is passed from the host with the faeces. If the excrement so infected be ingested by another *Nepa*, these resistant, oval, post-flagellate forms have their firm, varnish-like coat dissolved by the digestive juices of the host, and become capable of further development. Such forms are known as pre-flagellates. The pre-flagellates in the stomach of the *Nepa* gradually elongate. A flagellum arises near the blepharoplast, reaches the surface of the body at the anterior end and projects as a free flagellum.

By elongation of the posterior end of the body, the full, typical flagellate form is produced. Multiplication of the parasite occurs by longitudinal binary fission in both the pre-flagellate and flagellate stages. As the organisms pass onwards into the intestine, the environment is less favourable and they react accordingly. The body cytoplasm becomes concentrated, the flagellum is withdrawn and largely dissolved, and the now compact, more or less oval parasite secretes a varnish-like coat for itself, becoming thus the post-flagellate form again, in which condition it passes from the host. The life-history of *H. jaculum* in its insect host was fully described and illustrated by one of us in 1909.

While the anterior part of the gut of the *Nepa*, consisting of oesophagus and stomach, contains mostly pre-flagellate and young flagellate forms, the posterior gut consisting of the intestine

contains flagellates in various stages of division and the resistant, post-flagellate forms—a fact not without significance in our experiments.

III. *Experimental work.*

By a series of experiments we have established that *Herpetomonas jaculum*, normally parasitic in the Hemipteran, *Nepa cinerea*, can become pathogenic to mice when fed to them or when they are inoculated with it. We have also determined the form of the flagellate mostly responsible for the production of the disease and most capable of development within the body of the vertebrate.

The blood of the experimental mice and of the control mice was very carefully examined before they were used and no protozoa were found therein. Examination of the rodents' faeces, both before and during the progress of the experiments, was entirely negative. Some experiments were performed in Liverpool with mice and *Nepa* obtained locally, and others were carried out in Cambridge, the *Nepa* being collected from small ponds on Coe Fen. As the results obtained in each place were identical, the influence of climatic conditions and of possible variations in the strain of *H. jaculum* used were eliminated. As the mice used were mostly young, they and their controls were kept at or near body temperature.

In most cases an interval of a few hours was sufficient to allow of the appearance of rounded, non-flagellate forms of *H. jaculum* in the peripheral blood. The mice became weaker and either died or were killed *in extremis*. Examination of their organs showed the presence of non-flagellate (leishmaniform) and of flagellate forms in the liver, spleen, bone-marrow and blood, occasionally in other organs. The liver was always the seat of heaviest infection. The parasites kept the facies of *H. jaculum* and were pathogenic to the mice. The number of flagellate forms present and their fine development (see Pl. I) was noteworthy, and is unlike what has been obtained before in the experiments by Laveran and Franchini with other insect flagellates in mice.

Much time was devoted to the examination of the parasites in freshly drawn blood and in emulsions of organs taken at death. Smears of the organs were also fixed wet with Bouin's fluid or with osmic vapour followed by absolute alcohol. For staining, Giemsa's solution, haematoxylin and iron-haematoxylin, with or without eosin, were employed.

Experiment 1 (A. P.). Young, wild mouse, ♀, weight 3·3 grams, was fed once on teased anterior portions (oesophagus and stomach) of the alimentary tract of *Nepa cinerea*, infected with *Herpetomonas jaculum*, and was afterwards fed only on milk. The infected

feeding material contained pre-flagellate and a number of flagellate herpetomonads. Twelve hours after the infective feed, the mouse seemed ill, shivering, its coat staring, showing some difficulty in swallowing and passing much urine. It grew weaker, and died 50 hours after the first and only infective feed. Examination of the blood 12 hours after infective feed, negative; at 24 hours, free, rounded, non-flagellate (leishmaniform) elements seen; at 30 hours, a parasite within a leucocyte and a single flagellate were present in caudal blood. At autopsy the liver and spleen were hypertrophied, and kidneys, spleen and liver all very friable. Non-flagellate, leishmaniform bodies were found in the liver, spleen, kidney and lung smears. They were not numerous. A few flagellates were found in liver smears, and in still fewer numbers in those of the spleen. The control mouse, fed on milk only, died four days later. Its organs yielded no parasites on examination.

Experiment 2 (A.P.). Young, wild mouse, ♂, weight 3·2 grams, was fed once with the teased intestines of three *Nepa cinerea* infected with *H. jaculum*, and was afterwards fed on milk. The intestinal material contained mostly post-flagellate forms of *H. jaculum*, together with some flagellates, among which dividing forms occurred. Four hours after feeding, the blood contained a very few rounded, leishmaniform elements, while a flagellate was found in the blood 24 hours after the infecting feed. At 30 hours and 40 hours the same condition was found. At 40 hours the mouse became very weak, coat staring, eyes very dull. Some shivering occurred. It died 70 hours after the infective feed. The body showed some emaciation. The liver was slightly enlarged and contained a number of flagellates and a few non-flagellate forms. Some dividing flagellates also occurred. Both flagellates and non-flagellate forms were present in the spleen, which weighed ·03 gram and was friable, and also in the bone-marrow, though in smaller numbers than in the liver. A very few parasites occurred in the lungs and heart blood. The other organs were negative. The control mouse was killed five days later, and examination of smears of its organs was entirely negative.

Experiment 3 (A.P.). A young, wild mouse, ♀, weight 3·4 grams, was inoculated intraperitoneally with material provided by very finely teasing up the entire alimentary tracts of three *Nepa cinerea*, infected with *Herpetomonas jaculum*. The mouse was afterwards fed on milk only. Three hours after inoculation, one flagellate in metamorphosis to the oval, post-flagellate stage was found in the blood. At 12, 24 and 40 hours, a few leishmaniform parasites occurred in the blood. Fifty hours after inoculation the mouse seemed ill and died at 60 hours. The body was somewhat emaciated, the spleen and liver enlarged. Weight of spleen was ·04 gram. A very slight peritoneal exudate was present. Both

liver and spleen contained a few flagellate and non-flagellate forms. They were also present, though very rare, in the bone-marrow. No parasites were found in smears of other organs or in the peritoneal exudate. The control mouse, inoculated intraperitoneally with an equal quantity of normal salt solution, showed no parasites at all when killed.

Experiment 4 (H. B. F.). Young, wild mouse, ♂, weight 5 grams, was fed once with teased oesophagi and stomachs of three heavily infected *Nepa cinerea*. The feeding material contained pre-flagellate and young flagellate forms of *H. jaculum*. Ten hours after feeding, a very few non-flagellate forms were found in the blood. At 24 hours, they were still very few and the mouse was less active. Thirty-six hours after feeding, a rounded, leishmaniform non-flagellate was seen in a leucocyte. After this the mouse became steadily weaker and it was killed 60 hours after the infective feed. The liver was enlarged and also the spleen. The latter weighed ·05 gram. A few rounded, non-flagellate and flagellate forms occurred in the liver and spleen, they were very rare in the bone-marrow. No other organs were found to be infected. The control mouse, fed on milk exclusively, seemed quite healthy when it was killed 24 hours after the death of the experimental animal, and no herpetomonads in any stage have been found in its organs.

Experiment 5 (H. B. F.). A young, wild mouse, ♀, weight 6·5 grams, was fed once with the teased intestines of three *Nepa cinerea*, heavily infected with *H. jaculum*. In this case post-flagellate forms predominated, and many of the flagellates present were dividing. Six hours after the feed, a single, free, leishmaniform element was found in the blood; at 24 hours a few similar forms were found, and the same condition prevailed at 48 hours. At 60 hours a flagellate form was seen in the blood. The mouse also appeared to be ill, its coat staring and little food being taken. It was killed 84 hours after the infective feed, when it weighed 6 grams. The liver and spleen were enlarged, the latter weighing ·07 gram. The kidneys were more friable than usual. Flagellates were relatively numerous in the liver, somewhat fewer in the bone-marrow and spleen. A few dividing forms occurred. The leishmaniform stages were less common on the whole than the flagellate forms. The control mouse, fed on milk only, was killed by accident the day after the death of the experimental animal. Its organs contained no trace of herpetomonads or other parasites.

Experiment 6 (H. B. F.). A young, wild mouse, ♂, weight 8·5 grams, was inoculated intraperitoneally with the guts of two infected *Nepa* teased in normal saline, a control mouse being inoculated with a similar quantity of normal saline alone. Examination of the blood six hours after inoculation showed one

leishmaniform body. At 24 and 30 hours a similar condition prevailed. At 48 hours more leishmaniform bodies were seen, but the number always was very small. Seventy-two hours after inoculation, the mouse was very ill, so was killed. It weighed 7·5 grams. The spleen weighed ·08 gram, and like the liver, was enlarged and friable. The liver contained flagellate and post-flagellate herpetomonads, which were present, but in fewer numbers, in the spleen and bone-marrow. One flagellate only was seen in a preparation of the kidneys. The control mouse was killed on the day of the death of the experimental animal, and showed no parasites of any kind.

Experiment 7 (A. P.). An adult mouse, ♂, weight 17 grams, was inoculated intraperitoneally with the gut contents of three infected *Nepa cinerea*. Examination of its tail and ear blood showed a few leishmaniform parasites on the fourth day after inoculation, but no form of the *Herpetomonas* has been seen since. The mouse, after two months, is apparently well in health and has gained in weight.

Experiment 8 (H. B. F.). During a period of 36 days, a young puppy was fed with a total of 190 dog-fleas, *Ctenocephalus canis*, at nine feedings. The insects used were freshly drowned. Some of the fleas were infected with *Herpetomonas ctenocephali* (Fantham). At the first feed, the puppy weighed 1 lb. 4 oz., and at the last 3 lb. 8 oz. Ten days after the last feed, the puppy was feverish and ill, with a rectal temperature of 40° C., but rapidly recovered and is now normal. It has not increased so much in weight since the illness as the control puppy belonging to the same litter, *e.g.* when the control puppy weighed 10 lb., the subject of the experiment weighed 7 lb. 9 oz. Examinations of the blood made 21 days after the first and 12 hours after the sixth feed of fleas showed a very few leishmaniform elements both free in the blood stream; and, in one case, within a mononuclear cell. No flagellates have been seen. The puppy is still under observation, but at the present time (about four months after the beginning of the experiment) seems perfectly healthy and shows no parasites.

Herpetomonas ctenocephali then, like *H. jaculum*, is capable of living in vertebrate blood, *e.g.* in a dog, and of producing some organic disturbance.

We should not be surprised to learn later, that the so-called canine Kala-azar, occurring naturally in dogs in the Mediterranean region and stated to be spread by dog-fleas, is really a canine herpetomoniasis due to *H. ctenocephali* in dogs.

In connection with the practical import and significance of these experiments, it must be remembered that *Leishmania* develops into a flagellate, herpetomonad stage in the culture tube and in the gut of certain insects. Also, Franchini (1913) found a

herpetomonad, *Haemocystozoön brasiliense*, in a human subject and his findings have been accepted by Laveran.

IV. *The Morphology of* Herpetomonas jaculum *in the Mouse.*

A careful study of both fresh preparations and stained smears of the organs of the experimental mice has enabled us to trace the evolution of the parasite within the vertebrate. Very soon after the parasites have been introduced into the mice, the flagellate forms become rounded, their flagella disappear and they are found in the blood usually as free, leishmaniform bodies (Pl. I, Figs. 1, 2). Similar forms (Figs. 3, 4) occur in the organs of the mice, more particularly in the liver (Fig. 4) and spleen (Fig. 3). These forms generally show a distinct nucleus and blepharoplast (Figs. 1, 3, 4), while sometimes the blepharoplast is in contact with the nucleus (Fig. 2). Within the internal organs, after a period of rest, elongation may occur (Figs. 5, 6), some of the forms being stout (Fig. 5), others slender (Fig. 6). Each gradually elongates and produces a flagellum (Figs. 9—18), thus becoming typical herpetomonads. A short, deeper staining portion is first seen near the surface (Fig. 6), which reaches the exterior of the body, sometimes at right angles to the anterior end of the body (Figs. 9, 11, 12), sometimes obliquely (Figs. 10, 13). The nucleus may be karyosomatic (Figs. 11, 12), or the chromatin may be granular and more or less evenly distributed (Figs. 14, 16). The blepharoplast is usually bar-like (Figs. 12, 13), but is occasionally rounded (Fig. 9) or oval (Fig. 11). A basal granule may be present near the origin of the flagellum (Figs. 12, 16). Chromatoid granules are frequently present in the cytoplasm (Figs. 10—12, 14, 17). The posterior end of the body may be somewhat blunt (Fig. 12), or more or less tapering (Figs. 13, 15, 16, 18). The appearance of the flagellate forms lying between the cells of the liver (Fig. 15) resembles that of forms more or less separated from the cells, or present in the blood stream.

Multiplication of the parasite occurs in the mouse much as it does in the *Nepa*. Division occurs in non-flagellate forms (Figs. 7, 8), the division commencing with that of the blepharoplasts. Division of the flagellates has been seen in fresh preparations, and the various stages traced in stained smears. Fig. 19 shows one such stage.

It seems probable that within the mouse, some of the flagellates become oval, non-flagellate forms similar to those shown in Figs. 1—4. Such forms have been found in the liver, spleen, bone-marrow, heart, and, in one case, in the lungs of the mouse.

Some rounded, leishmaniform parasites have also been found within leucocytes (Fig. 20), though these were never numerous.

The non-flagellate forms measure 4 μ to 6 μ by 2 μ to 3 μ. The body of the mature flagellate varies from 3 μ to 32 μ, its breadth being 1·6 μ to 2·5 μ, while the free flagellum may reach 35 μ in length. The dimensions thus agree with those of *H. jaculum* in the insect.

We did not observe many small rounded elements, often uninucleate, and about 1 μ to 2 μ in diameter, occurring in red blood corpuscles of infected vertebrates, as described by Laveran and Franchini. Before discussing such endoglobular elements further we wish to conduct more experiments. Such elements do not occur in natural leishmaniases.

V. *The forms of* Herpetomonas jaculum *most infective to Mice.*

When the parasites are first seen in the circulating blood, they are usually in the form of small, leishmaniform bodies, and are usually free in the blood plasma. It seems very probable that the flagellate forms introduced into the mice gradually absorb their flagella and thus become non-flagellate forms which appear in the blood in exactly the same way as Fantham (1911) described for *Trypanosoma gambiense* and *T. rhodesiense* when inoculated into new hosts. When such rounded forms reached the internal organs, they appeared to undergo further development, and the parasites were always more numerous in the internal organs than in the circulating peripheral blood. In connection with the number of herpetomonads present in the hosts and the degree of illness produced in them, several features of interest have come to light as a result of our experiments.

The appearance of the parasites in the peripheral blood was almost as early in the case of mice fed with the intestines of *Nepa cinerea* infected with *Herpetomonas jaculum*, as in the case of those inoculated intraperitoneally. (See Expts. 2 and 3.)

The largest number of non-flagellate, rounded forms of *H. jaculum* occurred in the mice fed with the intestines of *Nepa cinerea*.

The largest number of flagellate *H. jaculum* also was present in the mice fed on the intestines of *Nepa cinerea*.

Very few flagellate and but few non-flagellate forms of *H. jaculum* occurred in the case of the mice fed on the fore-guts of the infected *Nepa*, while mice inoculated intraperitoneally showed an infection of flagellates considerably smaller than in the case of the mice fed on intestine, though slightly greater than those fed on the fore-guts of *Nepa*.

It must be remembered that the stages in the life-history of *H. jaculum*, as with other insect flagellates, vary in different parts of the insect host. While the fore-gut of the *Nepa* used contained non-flagellates (pre-flagellates) in a condition suitable for growth in the *Nepa* in which they were found, they were relatively non-resistant, and the flagellates present were not mature. On the other hand, the intestine of the *Nepa* contained flagellates, all of which were mature and some of which were dividing, and, in addition, the highly resistant post-flagellate forms, which are well adapted both for resisting unfavourable conditions and for adapting themselves to new environmental differences.

Thus we conclude that the post-flagellate stages of *H. jaculum* have greater powers of growing and multiplying successfully in the vertebrate host to which they were introduced. Also, these post-flagellate forms, together with the multiplying flagellates, were responsible for the greater number of flagellates found in their hosts, compared with those found in mice fed on the fore-guts of the *Nepa*. Consequently there seems evidence to show that the form of insect flagellate best adapted for transference from insect to insect, is also the form most effective in producing disease when introduced by way of food or intraperitoneally into the vertebrate, in the case of *H. jaculum* in mice. Probably the same will be found to be true in herpetomoniasis of the dog produced by swallowing dog-fleas infected with *Herpetomonas ctenocephali*.

VI. *Comparison of the forms of H. JACULUM found in the Vertebrate with those present in the natural Insect Host.*

There are striking resemblances between the forms of *H. jaculum* in the mouse and in *Nepa cinerea*. The non-flagellates present the same morphology in each case. The flagellates show practically the same structure. The main differences appear to be in details. Some of the very long forms of *H. jaculum* have not yet been found in mice, but they might be found were more experiments undertaken. The average size of *H. jaculum* flagellates in the mice and in the *Nepa* is the same. The herpetomonads in mice also appear to have somewhat more granular cytoplasm containing a rather larger number of chromatoid granules than do the corresponding forms in the insect host. In all other respects, the parasites keep the facies of *H. jaculum* as it occurs naturally in *Nepa cinerea*.

VII. *Comparison of the Induced Herpetomoniasis of the Mouse with Leishmaniasis.*

Some years ago, Patton described the formation of the flagellate form of *Leishmania* (*Herpetomonas*) *donovani* in the bed bug. While a flagellate stage of the Kala-azar parasite can develop in the bed bug, the non-flagellate stage appears to be the only one found in the vertebrate (human) host. In the induced herpetomoniasis of mice due to *H. jaculum*, the outstanding feature is the presence, in addition to rounded stages, of active, well formed flagellates in the liver and in less numbers in the spleen and bone-marrow of the host.

Non-flagellate elements, frequently termed leishmaniform, also occur in the vertebrate infected artificially with herpetomoniasis through *H. jaculum* or *H. ctenocephali*. These non-flagellate forms differ from the Leishman-Donovan bodies of Kala-azar in several respects: There is less uniformity of appearance, some of the non-flagellate *H. jaculum* or *H. ctenocephali* being much more elongate than is seen in the Leishman-Donovan bodies, while, what is more obvious, the position of the blepharoplast is different.

Typically, the nucleus of the Leishman-Donovan body is somewhat to one side, the deeply staining blepharoplast is immediately opposite to it, also to one side and lying transversely along the narrow diameter of the organism. The latter position is characteristic. The non-flagellate forms of *H. jaculum* in mice have their blepharoplasts in almost any position other than that characteristic of the Leishman-Donovan body. The position varies in parasites on the same smear from vertical to horizontal positions in any situation in the oval body. Again, free parasites seem less common in Kala-azar than they are in artificial herpetomoniasis.

That the two diseases are allied is undoubted. Both present the same features—the insidious onset, the subsequent relatively rapid illness, the splenic and often hepatic enlargement, feverish attacks and emaciation are common to both maladies, while in both diseases similar parasites are found.

VIII. *General Conclusions.*

From the results of our researches we conclude that:

1. Insect flagellates, *e.g. Herpetomonas jaculum* (Léger) from *Nepa cinerea*, and *Herpetomonas ctenocephali* (Fantham), parasitic in the dog-flea, *Ctenocephalus canis*, can live inside certain vertebrates (*e.g.* mouse and dog respectively) and can multiply therein. This we have shown experimentally.

2. If such flagellates be inoculated intraperitoneally or are fed by the mouth in food, the flagellates can find their way into the blood stream and internal organs (*e.g.* liver, spleen, bone-marrow) of the vertebrate host.

3. The insect flagellates are pathogenic to the vertebrates experimented upon, producing symptoms like those of leishmaniasis (Kala-azar).

4. The oval post-flagellate forms appear to be more capable of developing in vertebrate hosts than are other stages of the herpetomonad parasite of the insect.

5. It may be expected that the various leishmaniases, occurring in different parts of the world, will prove to be insect-borne herpetomoniases.

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Explanation of Plate I.

All figures were outlined with an Abbé-Zeiss camera lucida, using a 2 mm. apochromatic (Zeiss) objective and compensating ocular 8. The magnification is, in all cases, approximately 1300 diameters.

Fig. 1. Oval, non-flagellate form of *Herpetomonas jaculum*. From blood of mouse.

Fig. 2. Oval form of *H. jaculum*, nucleus and blepharoplast in contact. From bone-marrow of mouse.

Fig. 3. Rounded form with karyosomatic nucleus. From spleen.

Fig. 4. Typical, oval, leishmaniform element. From liver.

Fig. 5. Stout, elongating form. From bone-marrow.

Fig. 6. Slender, elongating form, root of flagellum seen. From lung.

Figs. 7, 8. Dividing, non-flagellate forms. From blood and from bone-marrow preparations respectively.

Fig. 9. Young flagellate with round blepharoplast. From lung.

Fig. 10. Flagellate with flagellum leaving the body obliquely. From bone-marrow.

Fig. 11. Herpetomonas with karyosomatic nucleus, and a number of chromatoid granules present. From bone-marrow.

Fig. 12. Larger flagellate with bar-like blepharoplast and basal granule present. From spleen.

Fig. 13. Similar flagellate with flagellum leaving the body very obliquely. From liver.

Fig. 14. Herpetomonad with chromatoid granules. From spleen.

Fig. 15. A group of three liver cells of mouse with one *Herpetomonas jaculum* lying between them.

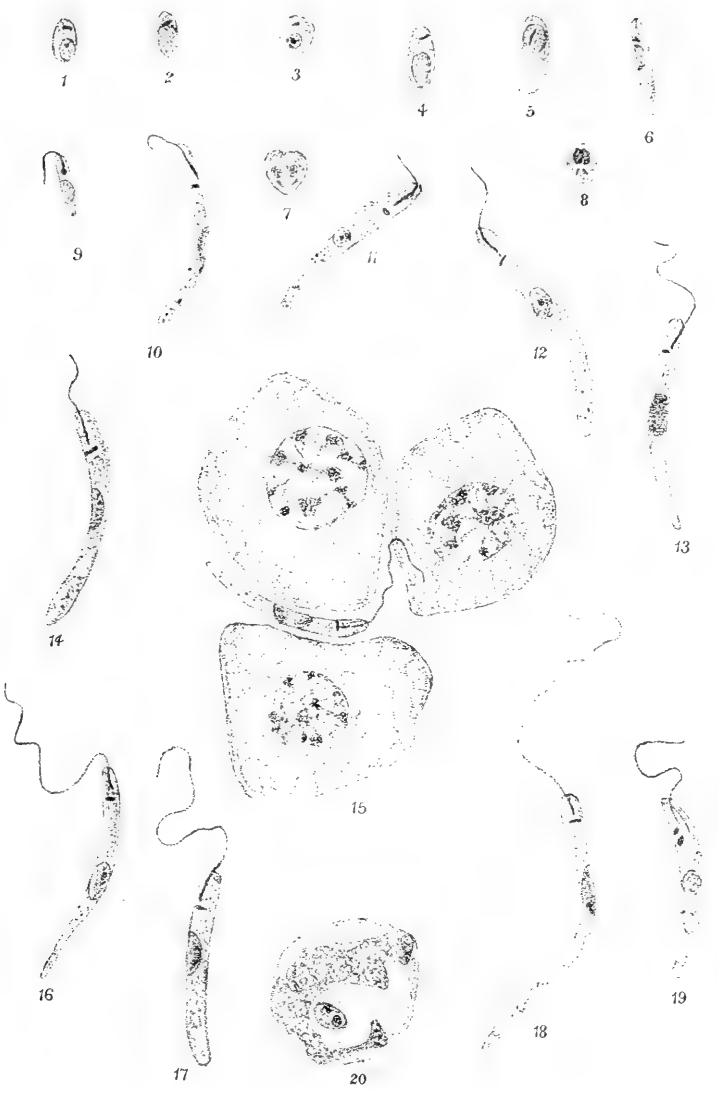
Fig. 16. Somewhat pointed flagellate, with evenly distributed nuclear chromatin. Small basal granule at base of flagellum. From the liver.

Fig. 17. Stout form with chromatoid granules. From liver.

Fig. 18. Slender, elongate flagellate. From liver.

Fig. 19. Early stage in division. The blepharoplast has divided into two, and splitting of the flagellum has commenced. From bone-marrow.

Fig. 20. Non-flagellate form in a leucocyte. Note the position of its blepharoplast. From circulating blood.



A.P. del. et lith.

HERPETOMONAS JACULUM IN THE MOUSE.

The Cuticula of Insects as a means of defence against Parasites. By WILLIAM R. THOMPSON. (Communicated by Mr F. A. POTTS.)

[*Read* 23 November 1914.]

A number of years ago, in his "Lessons on the Theory of Inflammation," Metchnikoff expressed his belief that the phagocytic reaction to parasitic organisms in Arthropods is in general rather feeble; and he suggested that this condition is correlated with the development of a chitinous cuticula which covers the entire body and lines a great part of the alimentary canal. Since this cuticular armour prevents the entrance of parasites, the stimulus for the development of the phagocytic function is absent. A little later, Cuénot, in his paper on the physiology of the Orthoptera, objected to this view, on the ground that the cuirass of the Arthropods, in spite of its thickness and its resistance, has never prevented the penetration of parasites; but that, on the contrary, it is just the animals of that group which are most heavily infested by internal parasites. The fact that normal parasites are not ordinarily molested by the phagocytes—which Cuénot had himself observed—he explained not on the basis of a feeble phagocytic power in the Arthropoda, but as the result of a progressive adaptation of the parasitic organisms which enables them to resist or repel the phagocytes by which they would otherwise be destroyed.

The question of the relation between the phagocytes and the parasites of the Arthropods is difficult and complex. Such data as I have been able to gather up to the present time seem to me to support the contention of Metchnikoff. At all events, toward the great majority of the internal metazoan parasites of Arthropods there is no visible direct phagocytic reaction. Without treating this matter in detail, I wish now only to consider the subsidiary question of the actual value to one group of the Arthropoda—the Insects—of the cuticula and its appendages as a means of defence against parasitic enemies.

In this connection the Dipterous and Hymenopterous parasites are especially interesting, for they ordinarily gain entrance to the body of their hosts through the integument. Those parasites which make their way into the haemocoele by the mid-intestine are naturally little affected by cuticular developments or functions.

The thickness and resistance of the cuticula in the larvae of Lepidoptera sometimes effectively prevents the entrance of the larvae of Dipterous parasites. I have frequently noticed hatched

eggs of Tachinid flies upon the bodies of insects which proved on dissection to be quite free from parasites. In the woods about Boston, Massachusetts, one often finds caterpillars of the Gipsy Moth (*Porthetria dispar* L.) bearing considerable numbers of the eggs of a native Tachinid parasite (possibly *Tachina mella* W.). Of a large number of such caterpillars which I once reared, less than one-half per cent. produced parasite larvae. In some cases I have observed—as C. H. T. Townsend has described—the young larvae of these Tachinids, just hatched, struggling vigorously to bore into the body of the Gipsy larva, but apparently quite unable to cut through the thick and heavily chitinized cuticula.

When the ovoviviparous Tachinid *Carcelia cheloniae* Rond attempts to oviposit upon the skin of its host, the caterpillar of the Brown-tail Moth (*Euproctis chrysorrhoea* L.), the thin-shelled eggs are rather often pierced by the fine bristles with which the nettling hairs of the host are beset, while the larvae which emerge from the uninjured eggs are sometimes impaled upon the barbs of the hairs while they are attempting to descend to the skin. Though these observations were made in the laboratory, I do not doubt that the same fate often overtakes the parasites in natural surroundings.

Upon several occasions I have collected Datanid caterpillars of which the head capsule and the dorsal shield of the last segment were almost covered by a mass of the large white eggs of some Tachinid parasite which I do not think I have ever reared. It is evident that larvae hatching from eggs in these positions very rarely succeed in penetrating into the body of the host. The explanation of this curious fact is very simple. Whenever the Datanid caterpillars are disturbed, they elevate the anterior and posterior segments so that the body is curved in a semi-circle, and in this position they remain for some time rigid and motionless. The result of this movement is the presentation of the anterior surface of the head and the supra-anal shield to any Tachinid which happens to be hovering above, and the parasite is thus misled into depositing its eggs upon the most resistant and impenetrable parts of the caterpillar's anatomy.

It is well known that by the process of moulting, which is in the most strict relation with the development of a chitinous cuticula, the larvae and nymphs of insects often succeed in liberating themselves from the unhatched eggs of Dipterous parasites.

Even when the parasite larvae have made an entrance into its body they may still be affected by the casting of their host's cuticula. In this way the caterpillars of the Brown-tail Moth sometimes rid themselves of the larvae of *Carcelia cheloniae*.

This parasite penetrates into the body of *Euproctis chrysorrhoea* through an opening which it makes in the skin of the caterpillar. Like many common Tachinid parasites it does not withdraw completely into the body cavity of the host, but comes to rest with its caudal extremity, bearing the respiratory stigmata, in the opening through which it has entered. The hypodermal cells in this region proliferate, at the same time actively secreting a considerable quantity of chitinous material; and there is formed a funnel-shaped "integumental sheath" surrounding the body of the parasite and opening at its ectal extremity to the exterior. I have several times observed that when the ecdysis of the host occurs a short time after the penetration of the parasite, the little larvae may be withdrawn from the body of the caterpillar in their integumental sheaths and cast off with the cuticle to which they are attached. Pantel has also found that at a more advanced stage, when the parasite larva is much too large to be "moulted out" by the host, the integumental sheath may break off at the point where it joins the integument, and slip down with the parasite which it surrounds into the general body cavity, where the Tachinid soon perishes from want of free oxygen.

As for the Hymenopterous parasites, they possess a well-developed apparatus—the ovipositor—by means of which in many cases they introduce their eggs directly into the body cavity of the host; and their larvae, more perfectly adapted to a parasitic existence than the majority of the Dipterous parasites, usually lie free in the haemocoele of the host, where they are not affected by the process of moulting. However, it is very likely that even against these parasites the cuticula is really a very important means of defence.

That the Arthropods support a vast number of internal parasites is undeniable. It may even be true, as Cuénot contended, that they suffer more severely in this way than any other group of the animal kingdom. In any case, however, it is not therefore safe to conclude that this heavy parasitism results from the inadequacy of the cuticula as a means of defence. The efficacy of a defensive structure can be determined only in relation to the vigour and extent of the attacks which it has to support. It appears to me that the question gains in interest if we turn it about and ask ourselves whether there is not something specially efficient about the parasites of the Arthropoda or some peculiarity of structure in the Arthropods themselves which makes them unusually susceptible to parasitic invasion.

One striking peculiarity of the Arthropods is that the parasitic forms in many cases prey upon other members of their own group. To a limited extent, it is true, the same thing can be said of other groups of animals. Some of the suctorial infusoria are

parasitic in Ciliates: the larvae of several of the Narcomedusae (*Cunina proboscidea*, for example) infest adult Trachymedusae. Cooke mentions three or four species of Mollusca said to be parasitic in other Molluscs, while the Hag-fishes and a few Teleosts are parasites on fishes. No doubt cases of this sort could be cited in several other phyla, but the fact remains that the vast majority of parasitic animals belonging to groups other than the Arthropoda—the parasitic Protozoa, Trematodes, Cestodes, Orthonectids, Dicyemids, parasitic Rotifers, Nematodes and Molluscs—practically all infest animals of other groups.

On the other hand, there are among the Crustacea, Arachnida and Insects, parasitic groups the members of which do not attack Arthropods—the parasitic Copepods, the Sarcoptids and the Oestrids may be cited. However, among the Crustacea, and especially among the Insects, an equal or much larger number of parasites invariably attack more or less closely related species. It will suffice in this connection to mention the Rhizocephala, the great group of the Epicarid Isopods, the Tachinids, Ichneumonids, Braconids and Chalcids.

Of the manner in which the parasitic habit arose among the Arthropods we know nothing. The fact that so many of them prey upon one another and that among the Insects true secondary and even true tertiary parasitism exists, seems to be one expression of the intense competition among the members of this truly dominant group. In any event it seems reasonable to suppose that forms which have taken to parasitizing their near relatives have many things in their favour. The life cycle of many parasites in other groups—such as the Trematodes for example—involves a passage from one environment to another, and sometimes includes two or more very different hosts. This is at best an uncertain business, and even a high rate of reproduction cannot altogether offset the chances of misfortune. With but few exceptions, adult Arthropod parasites inhabit the same environment as the Arthropod host, and comport themselves in it in a somewhat similar manner. No Arthropod parasite passes through more than a single host during the course of its life cycle. All of these features contribute to make these forms formidable enemies of the other Articulates upon which they prey.

It is also possible that the Arthropods in many cases are particularly well adapted to the needs of parasites. For example, Wheeler has suggested that the possession by the insects of great quantities of reserve material, utilized by the organism during the metamorphoses and in the development of the genital organs, and the exhaustion of which has but little effect upon the general vitality, renders these animals particularly fitted to support parasitic invasions.

I may also point out that the body cavity in Arthropods is of a rather special nature, especially in such forms as the larvae of the Insects, for it extends uninterruptedly from one end of the body to the other, and the organs can be subjected to considerable displacement in it without injury. Certain parasites are thus able to develop until they have attained a size almost equal to that of their host. Finally, as Mr F. A. Potts has suggested to me, the fact that the body cavity is a haemocoele is important, for parasites which develop in a haemocoele enjoy considerable advantages from the point of view of respiration and nutrition.

I therefore conclude that the very considerable parasitism to which Arthropods in general and Insects in particular are subject is not necessarily a proof of the inefficiency of the cuticula as a defensive structure. It appears to me more likely that this high rate of parasitism is due in part to the fact that a great many of the parasites are themselves Arthropods, in part to the Arthropod structure and life history which renders the members of this group especially able to support parasitic invasions. It seems highly probable that the cuticular armour, and the function of ecdysis correlated with it, in reality arrests a very considerable part of the violent attack which many members of the Arthropoda are obliged to sustain.

The Shortest Line Dividing an Area in a Given Ratio. By Norbert Wiener, Ph.D.* (Communicated by Mr G. H. Hardy.)

[*Received* 27 October 1914. *Read* 23 November 1914.]

The question we set out to answer in this paper is: given a simply connected area on a plane, what can we say, apart from any particular information we may have concerning the area, about the shape of the shortest segment of a curve, lying entirely in it, and dividing it in a given ratio, provided such a curve exists? To put the problem more concretely, let us suppose a farmer wants to divide an irregular field of his evenly between his two sons, and suppose he wants to use as short a hedge as possible. How shall he shape his fence? The conditions of the problem demand that the curve in question must have a length and be continuous. We shall limit our discussion in this paper to curves whose slope, considered as a function of the length of the curve from one end to the point where the curve has the slope in question, possesses only a finite number of discontinuities.

The method by which one would, at first thought, set out to solve this problem, would be that of the calculus of variations. But a little reflection will convince us that the condition that the arc dividing our area in a given ratio must lie entirely within the area, is difficult to express, and next to impossible to handle, by the methods of the calculus of variations.

Our problem is, however, easily amenable to an elementary treatment. It is easy to show that the line of our fence, for example, will be either an arc of a (finite or infinite) circle, or will be a chain of such arcs, such that two successive arcs only meet on the boundary of the area.

To demonstrate this I shall first have to prove the following lemmas.

Lemma 1. *Given a circle, and any two points on its periphery, then an arc of a circle can always be found passing through these two points, and dividing the circle in any desired ratio.*

For let the circle be called c and the two points A and B. Draw the chord $\overline{AB}$. Construct its perpendicular bisector, and let the latter meet c in the points C and D. Let E be a point on

* The following article is on a topic suggested to the author by Dr Otto Szász, *Privatdozent* at Frankfurt am Main. It was the author's original intention to have this article, with some further work of Dr Szász, appear under the joint authorship of Dr Szász and himself, but the war has rendered Dr Szász at least temporarily inaccessible, and this plan impossible. Dr Szász' work consisted in a rigorous demonstration that the shortest line dividing any *scalene triangle* in a given ratio is a circle with its most acute apex as centre.

$\overline{CD}$ between C and D. Let F be a point on $\overline{CE}$ between C and E. Then draw the circles AEB and AFB. The lune $ACBE$ is greater than the lune $ACBF$. For AEB and AFB only intersect at A and B, and E is outside $ACBF$. Moreover, by choosing E and F near enough together, we can make the lune $AEBF$ as small as we wish. For we can construct a circle concentric with AEB and passing through F. The ring between this circle and AEB will contain the lune in question, and will have the area

$$2\pi r\,(\overline{EF}) \pm \pi\,(\overline{EF})^2,$$

where r is the radius of AEB*. As $\overline{EF}$ decreases without limit, this will also. Therefore the area of the lune $ACBE$ is a monotone continuous function of the length of $\overline{CE}$ within the region from $\overline{CE} = 0$ to $\overline{CE} = \overline{CD}$. Therefore it can easily be shown by a continuity argument that

$$\frac{\text{area of } ACBE}{\text{area of } ADBE}$$

is a monotone continuous function of $\overline{CE}$, from $\overline{CE} = 0$ to $\overline{CE} = \overline{CD}$. and that in this region it takes every positive value.

LEMMA 2. *The shortest line passing through two given points on the boundary of a given circle, dividing the area of the circle in a given ratio, is an arc of a circle.*

Let our circle be, as before, c, and the two points A and B. By Lemma 1, there is an arc of a circle dividing c in the desired ratio: let it be AEB. If AEB be a segment of a straight line, our lemma needs no proof. If not, let AFB be any other curve dividing c in the same ratio. Complete the circle AEB, and let AGB be the other arc determined by A and B on this circle. Let ACB be the arc of c within the circle $AEBG$.

Then the area of the circle $AEBG$ and that of the figure bounded by AFB and AGB will be identical. For the two have the lune $ACBG$ in common, and, by hypothesis, the area of the lune $ACBE$ equals that of the figure bounded by AFB and ACB. By Steiner's theorem the perimeter of $AEBG$ must be less than that of $AFBG$, for it is a circle. Hence, since the two perimeters have AGB in common, the length of AEB is less than that of AFB. This proves our lemma.

Our theorem is now easy enough to prove. For let us suppose our area given, and the shortest line dividing it in a given ratio also given. Let us call the latter l. From any point on l at a positive distance from the boundary as a centre, we can describe

* If AEB is a straight line, then $AEBF$ may be enclosed in a rectangle whose base is constant, and whose altitude may be made as small as you will.

a circle lying entirely within the area. Except, at the most, in a finite number of points, we can make this circle small enough to cut l in two points only*. Within the circle, l must be an arc of a circle. For, call our little circle c. Let l divide c in the ratio $\frac{m}{n}$. Construct the arc of a circle cutting c in the same points as l, and dividing c in the ratio $\frac{m}{n}$. Then consider the curve formed by this arc and the portions of l outside c. This must divide the area in the same ratio as l, and, if it is not the same curve as l, must be shorter. This contradicts our hypothesis.

In the same way, it may be shown that l cannot contain two arcs of distinct circles meeting inside the area. For, as before, about the meeting-point of these arcs describe a circle, c, cutting each arc in one point only, and lying entirely within the area. Then, by the same reasoning as before, the portions of the two arcs lying within c must form a single arc, which is impossible. Thus our theorem is proved†.

* This is a consequence of the condition which we laid down for all curves considered in this article—that the slope of the curve at a point p, considered as a function of the length of the curve between p and one of the end-points of the curve, possesses only a finite number of discontinuities. This is at once obvious, if we reflect that any curve $y = f(x)$, which intersects a circle c described about some point on it more than twice, must have a maximum or a minimum in y between one of the points where it intersects c and the centre of c.

† It is almost self-evident that the shortest line to divide a *convex* area in a given ratio is a *single* arc of a circle, but this I have not yet been able to prove.

The Colour Variations of the Fauna associated with Crinoids.

By F. A. Potts, M.A., Trinity Hall.

[*Read* 23 November 1914.]

Although a number of animals have been previously described as living on crinoids and matching their hosts in colour no complete study of this commensal fauna has been made. During a visit to Murray Island, Torres Straits, in October 1913, I was able to make some observations on the wonderfully rich crinoid fauna on the reefs there. The commonest species is a form called *Comanthus annulatum* Bell remarkable for its extraordinary range of colour variation from very light coloured individuals in which white, light green, yellow and grey mingled in the colour scheme to others which are entirely dark green or black. In the shelter of its arms live commensal forms belonging to many groups of marine invertebrates and generally speaking they possess a type of coloration which makes them inconspicuous upon the host and so varies with the colour of the host.

The commensals so far as they have been identified are given in the following list:

Crustacea.	Decapoda Macrura. *Synalpheus comatularum* Haswell, *S. Brucei* sp. n.*, *Periclimenes* spp. n.† and a new genus (*Pontoniopsis*) and species of Pontoniid prawns‡.
	Decapoda Anomura. *Galathea longirostris* Dana and two new species of *Galathea*.
	Isopoda. *Cirolana* sp. n.
	Amphipoda. A new genus of the family Amphilochidae.
Echinodermata.	Ophiuroidea. *Ophiomesa cacaotica* and probably other species of brittle stars.
Annelida.	Polychaeta. A form probably near *Polynoë minuta* Potts.
	Myzostomidae. A number of species.

The different commensals have responded in different degrees to the stimulus causing this colour resemblance to their host. Thus among the Crustacea, in *Synalpheus* all stages of variation are met with, according to the individual inhabited, between a pale form with very narrow stripes of pigment to an extreme form

* Probably the whole *comatularum* group of the genus *Synalpheus* are associated with crinoids.

† A number of species of *Periclimenes* have been described from crinoids.

‡ Mr L. A. Borradaile is publishing a description of *Pontoniopsis* and the new species of *Periclimenes* in a forthcoming revision of the Pontoninae.

totally covered with dark pigment, while in *Cirolana*, on the other hand, the individuals associated with even the darkest crinoids, possess only insignificant lines of pigment on the otherwise totally white body. In the latter case then there is no protective resemblance although we witness the incipient stages of its establishment. Other forms like the polychaet, the amphipod and the brittle star *Ophiomesa* only possess dark varieties and occur upon dark green or black crinoids.

The most constant commensals of *Comanthus annulatum* are the two species of *Synalpheus*. *S. Brucei* was found in great numbers

Fig. 1. *Synalpheus comatularum* Haswell. ×2. To show colour pattern. *ch.*[1] smaller chela, the finger of which is modified for temporary fixation to the host.

on the crinoids from the reef at Murray Island but at depths of about five fathoms it is largely replaced by *S. comatularum*. In both species the habits of the Alpheid and its colour variations are similar. Each crinoid harbours a pair, male and female, which sit side by side on the disc, with their heads directed toward the mouth. When disturbed they cling firmly to the host, in *S. Brucei* digging the apical spines of the thoracic feet into the soft flesh of the disc while in *S. comatularum* the extraordinarily modified thumb of the smaller chela is hooked into the flesh or round a pinnule of the crinoid. When hunted off the disc they seek refuge on the under surface of the arms and when detached from the crinoid speedily return to its shelter. In addition they are remarkably thigmotropic, gathering together in an inextricable mass when kept in the same vessel.

The majority of the crinoids are dark green or black in colour and most of the commensal Alpheids are completely covered by a black or brownish-purple pigment. But often the pigmented dorsum is traversed by longitudinal stripes which are free from pigment, one down the middle line and two or three on each side. The relative extent of pigmented and nonpigmented areas is exceedingly variable and corresponds roughly to the coloration of the host which the Alpheid inhabits. Individuals lodging on crinoids whose general hue is light green have only thin purple stripes the rest of the body being unpigmented. In those members of the species *S. Brucei* which frequent a second host, *Comatula pectinata*, in which a bright red pigment predominates, the crustacean was also observed to show a red pigment covering the whole body. The range of colour variation is thus in *S. Brucei* limited between these two related types the red and purple; but by an adjustment of the relative extent of the bands of colour the animal may become inconspicuous upon the host whatever the colour scheme of the latter may be.

In most cases both commensals on any crinoid are similarly coloured: rarely however there is a marked difference. I may mention one case in which the one member was marked with very definite and fairly wide stripes of dark pigment (the intermediate non-pigmented areas being prominent) while the other was uniformly covered by red pigment. I am inclined to explain this and all cases where the Alpheid is rather conspicuous upon its host (as occurs in a certain proportion of individuals) by supposing it to have migrated from some other crinoid at a comparatively recent period. So thickly do the crinoids lie in the crevices of the reef, all conceivable colour-varieties being herded together without distinction, that it is more than likely that an interchange of commensals should occasionally take place for *Synalpheus*, though tending to become a truly sedentary animal, is yet very active and an excellent swimmer.

In discussing generally the protective resemblance of *Synalpheus* to *Comanthus* a comparison may profitably be made with the classical case of *Hippolyte varians* described by Gamble and Keeble. The young *Hippolyte* is free swimming and colourless but it becomes virtually a sedentary animal anchoring itself to a seaweed in the Laminarian zone on which it finds both food and shelter. The prawn has the power of forming red, yellow and blue pigments and by altering their relative proportions in the chromatophores it can acquire a green, brown, blue or red ground colour and is thus able to adapt itself to the varied colours of the seaweeds and hydroids. The pigment may be laid down in longitudinal stripes or horizontal bars and in this way a colour scheme can be formed matching whatever seaweed the prawn shelters in. In early life

a change in habitat is followed by a readjustment of the pigment altering the colour scheme but this power is soon lost.

The case under discussion is similar in kind though not in degree. *Synalpheus* is capable of producing only the two types of pigment—the red and purple. These moreover are only laid down in the one pattern in which longitudinal stripes are the dominant note. The paucity of pigments points to a limitation in the power of variability of the animal—but at the same time by alterations in the width of the stripes an approximate protective effect is brought about. This effect is plainly a direct response to the action of the environment. The general character of the colour pattern is plainly fixed for both species: possibly by natural selection since most of the other crustacean commensals show the same longitudinal striping. It is however difficult to see how natural selection can have brought about the power, in the individual, of responding to the colour scheme of the host by developing the requisite quality and quantity of pigment. Moreover in this case it would seem that even without a close resemblance *Synalpheus* and the other commensals are adequately protected by their association with the crinoids. For on the approach of predatory animals the long arms of the crinoid close over and so defend any commensals which happen to be on the disc from threatened attacks. These arms principally composed of calcium carbonate would be a disagreeable mouthful for any fish and I never met with crinoids with mutilated arms. Moreover the Alpheids are exceptionally active and can thus easily evade any unwelcome attentions.

The commensal Galatheids are much rarer but they too show an alternation of vertical bars of purple pigment with non-pigmented areas. The new genus of prawns, *Pontoniopsis*, also shows a combination of the striped pattern with yellow (or green) and brown pigments which makes it comparatively inconspicuous on the light coloured crinoids it inhabits. The Myzostomids may exhibit radiating bars of pigment different to the ground colour—their resemblance to the host is sometimes very striking. But in the other commensals the striping is not very marked or entirely absent.

It is possibly significant that the larger forms, for which protection is more necessary, are generally speaking those which show the greatest colour variability and the latter feature may be responsible for the almost universal occurrence of *Synalpheus* upon crinoids in the Torres Straits area.

A full account will appear in a forthcoming publication of the Department of Marine Zoology of the Carnegie Institute of Washington.

Preliminary notes on some Problems connected with Respiration in Insects generally and in Aquatic forms in particular. By G. L. PURSER, Coutts Trotter Student, Trinity College. (Communicated by Mr F. A. Potts.)

[*Read* 23 November 1914.]

THE means by which the gases are carried to and from the respiratory organs, in the majority of Triploblasts, is the blood, which is a general carrier for all substances to be used or got rid of by the cells. These substances are usually simply dissolved in the serum, but oxygen, being often required in larger quantity, is also carried loosely combined with some substance which is situated in the serum or in special respiratory corpuscles. The two substances known to be respiratory in function are the pigments Haemoglobin and Haemocyanin, the former an Iron compound of a globulin, the latter a Copper one of like form.

When, however, we examine the respiratory apparatus of an aërial Insect we find an altogether different arrangement, the reason for which is rather obscure. We do not, unfortunately, know from what stock or stocks the Insecta arose and have, therefore, little idea what the primitive insect was like, but still the origin of the tracheal system is a subject upon which it is interesting to speculate.

Examination of the respiratory organization of a group containing both aquatic and aërial forms brings out the fact that the general method for aquatic members is evagination as a gill and for aërial members invagination as a lung, as for instance in Vertebrata. Fish have gills and the terrestrial members have lungs. Again in the Mollusca aquatic members of the Phylum, such as the Lamellibranchiata, have exposed gills, while in aërial members the mantlefold, used by so many aquatic ones to protect the gills, encloses a chamber which acts as a typical lung.

Similarly in Arthropoda the aquatic forms, most Crustacea, have evaginations, gills, while the aërial forms, most Insecta, have invaginations, tracheae. The organ of respiration is therefore of the type that we should expect from an examination of other phyla.

Another characteristic, which might well be expected in Arthropoda, is the serial repetition which is exhibited. Metameric Segmentation, which lies at the basis of the Arthropod Classification, is exhibited by almost every kind of organ in the animal's body. Hence, what could be more natural than that the respiratory organ should follow suit and be serially arranged also?

Where the principal difficulty arises, however, is in attempting to explain the most peculiar feature of the whole arrangement, the extraordinarily fine ramifications of the tracheal system into the deepest tissues, with the result that the blood has been deprived of one of its most important functions, the carriage of oxygen from the respiratory organ to the cells requiring it and the excretion of CO_2. The reason for the origin of the haemocoele would perhaps help us towards the solution of this difficulty but unfortunately no adequate explanation is forthcoming. On the supposition, however, that the Insects arose when the haemocoele had only just begun to enlarge, we can quite easily imagine the course of evolution. The respiratory organs at that time would be simple pockets of ectoderm and cuticle serially arranged along the body. As the haemocoele enlarged the rate of circulation would most likely decrease. This would stimulate an increase in the surface of the tracheal system, owing to the need of a higher oxygen concentration in the blood, and of a better excretion of CO_2. Anastomosis and further ramification would follow until we get, as a final result, the oxygen being delivered straight to the cells requiring it in the gaseous state. All this, however, is pure speculation and depends upon the assumption, which is perhaps unwarrantable, that when the Insecta became differentiated from the rest of the Arthropoda the haemocoele was of quite small size, or, at any rate, that the circulation was very much faster than at present.

When Insects took to aquatic life [for the consensus of opinion at the present day is that aërial forms gave rise to aquatic forms rather than vice versa] their method of respiration was one, if not the chief, of their physiological difficulties. This same difficulty has arisen in other groups, the Mammalia, for example, where the Cetacea have become marine. These, however, have shirked the difficulty, and take long breaths, having, therefore, to come to the surface at intervals to renew their supply. Similarly many aquatic insects have to come to the surface to breathe. These, for want of a better term, I call False Aquatics. The methods by which they obtain their supply of oxygen and prevent the entrance of water into their tracheae are extraordinarily diverse and make a delightful branch of study, but since they do not differ physiologically from aërial forms I will say no more about them. The other division into which I divide aquatic insects is, in contradistinction from False Aquatics, True Aquatics. These have, in one way or another, become specialized to make use of the oxygen dissolved in the water in which they live.

On examination of aquatic insects generally we find that, though many false ones are imagines, none of the true group are. This is an interesting point because it seems to show that the need

for aërial flight has not been lost yet in any form, and that although such important changes of an adaptive character have taken place in the larvae the adults have so far been unaffected. The larval stage is the one of growth and therefore more dependent on its surroundings than any other. Consequently it is all the more likely to be modified by any alteration in those surroundings.

This modification, it is important to remember, is very different in closely allied forms. Let us take the case of *Culex* and *Chironomus*. The imagines of these two Diptera are almost identical and are distinguished from one another in the field by the way they hold their legs when at rest, yet the young stages of *Chironomus* contrasted with those of *Culex* would place it almost as far away as another Order; for these reasons: the larva of *Culex* comes to the surface and breathes by a pair of spiracles which are situated at the end of a respiratory syphon on the eighth abdominal segment. It is an active larva, living at the surface of the pool with colourless blood and a well-developed tracheal system; in fact a typical false aquatic larva. The larva of *Chironomus*, on the other hand, lives nearly the whole of its life in a tube in the mud at the bottom of a pool, has its blood deeply pigmented with haemoglobin, and breathes by blood gills which are tubular outgrowths on the last two segments, the tracheal system being almost if not entirely absent.

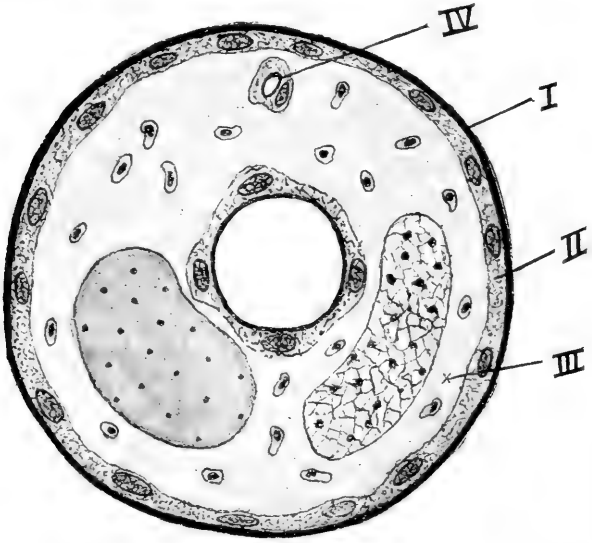
When we examine the pupae of these two we find that respiration is effected in *Culex* by means of a pair of trumpets which come off from the thorax and open to the atmospheric air, whereas in *Chironomus* it is effected by a pair of respiratory tufts arising from the same part of the pupa as in *Culex*, which are true aquatic respiratory organs, i.e. they make use of the oxygen dissolved in the water.

This divergence which has occurred between closely allied larvae has resulted in convergence between larvae of insects very widely separated phylogenetically. A good example is found in the comparison of the larvae of *Gyrinus*, *Sialis* and some of the Trichoptera. These in general shape and structure of respiratory organs are extraordinarily similar, but no one would say that the Coleoptera, Neuroptera and Trichoptera are as closely related as this semblance suggests.

When we consider the changes in the physical state of the oxygen as it passes from the surrounding medium to the cells, the difference between the respiratory methods mentioned above is emphasised. In the larva of *Chironomus* it remains in solution all the time; in normal insects and false aquatics it remains in the gaseous condition until it reaches the cells needing it; but in the true aquatics with tracheal respiration it is taken out of solution in the gills and carried through the body as a gas.

This last change of state is very remarkable owing to the difficulty of providing a physical and physico-chemical explanation of it. For a discussion of this subject I will refer to Miall in his admirable book on the *Nat. Hist. of Aquatic Insects*, page 37.

Now let us examine the microscopical structure of a typical simple tracheal gill, for instance, such an one as the larva of *Sialis* possesses. These gills, like in so many other forms, are arranged one pair to each abdominal segment for at least the first few segments, seven in the case of *Sialis*. The last segment bears a single filament which resembles two ordinary gills fused, but differs from them in being unjointed. This jointing is a feature peculiar to *Sialis* and is absent in those simpler forms found among the Trichoptera.



Diagrammatic transverse section of Gill of *Sialis*.

In trans. sect. they show the following structures (see fig.).

(I) Outer chitinous covering, the exoskeleton.

(II) The epidermal layer of cells, which secrete the exoskeleton. I shall refer to this as the hypodermis.

(III) The Haemocoele, in which may be seen, if the section be taken near the base, muscles and portions of Fat-body; and wherever it may be taken

(IV) The Tracheal Tubes of chitin covered by their layer of epithelium. One main tube, with small side branches running to the periphery.

These gills are circular in trans. sect. but taper longitudinally. During life they are curved upwards and backwards over the segments of the abdomen.

The Larvae of *Gyrinus* and Trichoptera, which we mentioned above as being so like that of *Sialis* in general form, will bear the comparison being carried as far as the microscopical structure of their gills. There are minor differences, for instance in *Gyrinus* they are fringed, and in some species of Trichoptera the main tracheal tube is double, but the essential structure is identical.

The Ephemerid larvae have gills which agree with those just described in being segmentally arranged, but differ from them on two points.

(i) Though the gills may be much subdivided they are fundamentally lamellate and not filiform. There are many grades of subdivision; in some species there is simply the double blade, quite lobose in character; in *Chloëon* the lamella tapers distally to a filament; while in *Caenis* the lamella has a fringe on its margins.

(ii) They contain a brown-black pigment. If a gill of *Chloëon*, which has been mounted unstained, be examined under a hand lens, the whole of the tracheae are visible as black lines spreading over the lamella and a single one can be seen passing up the filament mentioned above. On microscopical examination this coloration is found to be due to a brown-black pigment which is present in a finely granular state in the cells of the tracheal epithelium. This, however, is not always its position. In *Caenis* it is situated in the hypodermis. Hence it could be found all over the body surface. Examination of serial sections of the abdomen shows that its distribution is restricted (i) to the gill surfaces, (ii) to the lower surface of the anterior pair of gills, which act chiefly as shields to protect the others, and (iii) to that part of the general body surface which is also protected by the anterior gills; in fact it is found in practically the whole of the hypodermis of the "Branchial chamber" and nowhere else. This distribution in the individual and its occurrence in other aquatic insects in analogous, but certainly not homologous, organs, suggest the hypothesis that this pigment, which we will call "Spadicin" [Lat. spadix, dark brown], is a respiratory pigment. There are a number of objections to this. First of all, its state; the solid condition not being a very active one usually. It may be urged, however, that some substances, Platinum for instance, are very powerful chemical agents in a finely divided state.

Again, its absence in many forms where we should most certainly have expected it, in *Sialis*, *Gyrinus*, etc. This objection is difficult to cope with owing to our lack of knowledge of all these larvae, but in *Sialis*, perhaps, the blood takes its share of the oxygen, and so the rapid interchange which necessitates the pigment in

connection with the tracheal system is to a large extent done away with.

Some direct evidence was sought by subjecting larvae to asphyxiation. Since both Haemoglobin and Haemocyanin exhibit a colour change when reduced it was hoped that Spadicin would show one also. As far as experiments have gone no such result has been obtained.

There is very little of the general chemistry of Spadicin known. It is apparently insoluble in water and alcohol, the latter up to boiling point, but it is decolorized (dissolved ?) by strong Potash. During the reaction there is a colour change from brown-black through reddish to almost if not complete colourlessness. Owing to the difficulty of obtaining sensible amounts of the pigment, tests are usually unconfirmed and the results must therefore be received with caution.

The great point in favour of Spadicin being respiratory is its distribution. It is found in the respiratory organs, gills and tracheal tubes in Ephemeridae and Odonata. If these organs were homologous this would not be a very strong point but they are most certainly not. In Ephemerids they are tracheal gills arranged segmentally; in Agrionids they are also tracheal gills but compose a group of three round the anus; while in *Aeschna* the rectum is modified to form a respiratory organ. Why is it that Spadicin only appears in the respiratory organs of true aquatic insects if it has no respiratory function?

Now let us examine the respiratory organs of the Odonata:

The Agrionid larvae have three lamellae at the posterior end of the body with its tracheal system spreading over the surface like the veins of a leaf. The pigment, as far as I can tell, is the same as in the Ephemerids and the same remark applies to *Aeschna*. The Agrionid larval tracheal gills, however, cannot be considered very important or very useful in connection with a discussion of the question of respiration, owing to an extraordinary lack of definite knowledge about their function. It is thought by many that they are not respiratory at all, but purely locomotory, being analogous to the uropods and telson of the Decapod Macrura, and although I cannot go that far owing to their structure being so typically that of a gill, it is significant that they seem to be able to live without them just as well as with them.

The rectal wall of *Aeschna* has six thick longitudinal bands separated by thin and flexible membranes. Each longitudinal band bears a double row of transverse folds which enormously increase the epithelial surface and at the same time lodge the tracheal branches. Oustalet estimates that there are no fewer than twenty-four thousand of the folds. The tracheal branches enter larger, regularly arranged air-tubes which in turn open into the main

trunks running the length of the body. A large volume of water is sucked into the rectum at pleasure, and from this the tracheae draw a fresh supply of oxygen. The vitiated water can be expelled either gently, or, when the larva requires to propel its body forward, with considerable force. The above account is taken from Miall who unfortunately does not give Oustalet's reference. He goes on to say that these larvae, under strictly natural conditions, may increase the effectiveness of their respiratory organs by coming to the surface and taking in air directly. This would be made use of particularly if the water became foul.

The pigment, as far as can be made out from the sections already examined, is not situated in the tracheal epithelium, but in the hypodermis. If this is the case, we have in the Odonata just the same variation in the position of Spadicin as there is in the Ephemeridae. Whether there is any significance in this fact, that though it may be found in one of two layers of epithelium, it is never, as far as I know, found in both in the same species, we have not the slightest hint, but it is one of the many points which a physico-chemical explanation of the action of these gills will have to elucidate.

Of all the methods of respiration found amongst these true aquatic insects, the larvae of *Chironomus*, and perhaps others, exhibit the one most divergent from that of a typical insect.

Whether they live at the surface or down in the depths of the pond they breathe by blood gills on the last two segments, two pairs of finger-like gills on the penultimate segment, and a rosette of four on the last. The only tracheae to be found in the fullgrown larvae are a few twigs in the thorax. The blood of these larvae performs, therefore, all the functions it is called upon to do in all groups of animals possessing a vascular system, with the exception of the Tracheates. In those species of *Chironomus* which live in holes in the mud their blood is highly pigmented with haemoglobin. This acts most likely as a storehouse of oxygen between two excursions to the upper layers of water.

Before bringing this paper to an end I must again refer to Spadicin because it has been suggested that its function is excretory. This brings us face to face with the following problem. In a larva living in the air with its numerous stigmata giving free communication between the tracheal system and the exterior, the lining of most of the tracheae is pulled out at each ecdysis, and a new chitinous tube of rather larger size is laid down. Thus the growth of the main tubes is allowed for and the useless lining got rid of. But what happens when such a larva as *Chloëon* undergoes ecdysis? If the lining is pulled out at this stage then the stigmata must open for a short time. This requires that (i) the openings be closed immediately after ecdysis or else the creature

would drown, and (ii) that there is an attachment between tracheal system and exoskeleton at base of gills, the only place where the tubes could be extracted. These two requirements have not been observed to be present and in cast skins of Agrionid nymphs no signs of tracheal system is found. Spadicin (then) cannot be excretory, in the sense that it will be cast out with the tracheal lining, but it may be excretory in the sense that it is a break-down product of the old chitinous tube, which has been absorbed.

Another suggestion is that it is the enzyme or proenzyme for the absorption of the chitin.

Up to now no work has been done expressly to test these suggestions, but they seem unlikely because of the distribution of the substance. It would be found on all the tracheae in all apneustic larva [not in the gills and main trunks of some only], and with regard to its having anything to do with the absorption of chitin the case of *Caenis*, and perhaps of *Aeschna* also, puts that out of court. But although Spadicin may have nothing to do with the problem, the question of ecdysis and growth of the tracheal system of apneustic larvae is one of the most important which the study of these aquatic forms raises.

This is the position up to the present. By careful examination of as many new forms as possible, and from the experiments which Mr Balfour Browne, to whom I am very grateful for much advice and valuable criticism, is conducting on Agrionid nymphs, I hope soon to have sufficient new facts and observations to decide, to my own satisfaction at least, some of the numerous questions which the examination of the respiratory organization of true aquatic insects has raised.

On the Conditions of Instability of Electrified Drops, with Applications to the Electrical Discharge from Liquid Points. By JOHN ZELENY, B.A., Ph.D., Professor of Physics, University of Minnesota, U.S.A. (Communicated by Professor Sir J. J. Thomson.)

[*Read* 9 November 1914.]

IN a recent paper on the electrical discharge from liquid points*, a description is given of certain motions which are observed at the surfaces of such points when an electric current starts to flow from them.

The present paper is concerned with a study of this surface behaviour under a variety of conditions, and an explanation of the phenomena is given.

Description of Experiments.

1. A discharge point of the kind in question consists of a small drop of liquid at the end of a tube, usually only a fraction of a millimetre in diameter. The drop may be made to assume a hemispherical form by applying a suitable pressure to the liquid inside of the tube.

The arrangement of the apparatus used is shown in diagram in Figure 1. The discharge point A was made of glass or quartz and the current from it passed to the brass disc B (usually placed at a distance of 1·5 cms.) and then to earth through the galvanometer G. The reservoir C is joined to the bent glass tube D by the rubber tube E, which permits the reservoir to be raised or lowered for adjusting the pressure inside of the drop of liquid at the end of A. The liquid in the tubes is connected, by the wire F, to the source of potential and an electrostatic voltmeter.

In many of the experiments to be considered the essential parts of the apparatus were enclosed in a metal vessel to permit of work with various pressures and with different gases.

Unless otherwise stated, it will be understood throughout the paper that a positive discharge from the point is being considered.

2. When water is the liquid used in air at atmospheric pressure, it is noted, when the end of the point A is observed with a microscope, that as the potential is gradually raised, the surface of the drop being kept hemispherical by constant adjustment of the reservoir C, finally at a certain voltage the surface of the drop suddenly snaps back into a more flat position and the galvanometer indicates a momentary current. After adjusting the pressure so as to bring the surface to its first position, the phenomenon is only

* J. Zeleny, *Physical Review*, N.S., Vol. 2, p. 69, 1914.

repeated after the potential has been increased above its former value. On continuing the procedure, the liquid surface soon takes on a continuous up and down motion, a current of like period passing to the plate. At last the oscillations become too rapid to follow and the top of the meniscus simply appears blurred. On increasing the voltage further, the outline of the meniscus suddenly becomes perfectly distinct and the current becomes steady, a telephone receiver in the circuit now giving no sound. At this stage of the discharge the luminosity is confined to a very thin coating on the liquid surface, whereas in the intermittent stage a faint light was observed extending, in the form of a brush, a good portion of the distance to the plate.

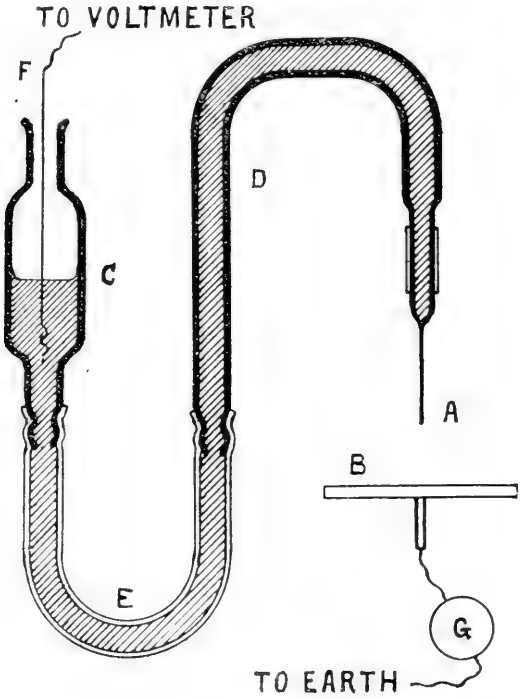


Fig. 1.

3. Experiments with water surfaces in air at reduced pressures show that the intermittent stage of the current is not obtained at pressures below about 60 cms. of mercury. In this region the current begins with a jump to a value of some magnitude, the meniscus of the liquid snapping back simultaneously to a lower

position; but the current may be maintained at a lower voltage than that at which it began.

4. A water surface in carbonic acid at atmospheric pressure shows the same behaviour at the lower potentials as that described in § 2 when air was used, but as the voltage is raised the blurred meniscus suddenly changes in this case not to a distinct hemispherical form as before but into a steady cone with a faint dark line extending outward from the apex for some distance, as shown by the drawing "*a*" in Fig. 2.

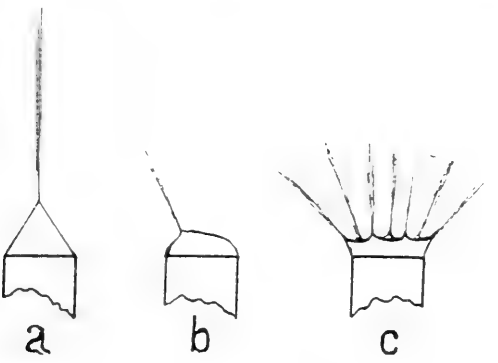


Fig. 2.

The height of the cone changes with change in the liquid pressure and when this is diminished sufficiently the form becomes that shown at "*b*" in the figure, where the small pointed portion at one side is to be noted as well as the faint dark streak extending out from it. The cone-shaped meniscus persists over a rather narrow range of voltages, and when the upper limit of this range is reached the surface again becomes agitated.

5. The potential at which the current ceases from a water surface in carbonic acid was found to be almost if not exactly identical with that at which the current ceases from the same point in air. Carbonic acid, however, shows the unique behaviour that this voltage remains unchanged for pressures down to about one-third of an atmosphere, below which pressure the intermittent stage of the current disappeared. The conical meniscus was not obtainable for pressures much below the atmospheric pressure.

6. In hydrogen no intermittent stage of the current was observed with a water surface whatever the pressure of the gas (below an atmosphere). The current begins suddenly with a considerable magnitude, as described in § 3 for air at reduced pressures.

7. An alcohol surface, both in air and in carbonic acid, was found to behave in the same way as a water surface does in carbonic acid, both as regards the oscillations with an intermittent current, and as regards the subsequent conical form of meniscus. For voltages well above that for which the steady conical surface changed into a blurred agitated one, the appearance became that shown at "*c*," Fig. 2, where there are a number of fine points with their attendant dark streaks arranged along the circumference of a raised edge.

For voltages at which the oscillations of the meniscus first appeared, these oscillations had a much smaller amplitude with alcohol than they did with water, and the quantity of electricity reaching the plate at each movement of the meniscus was also much less in the case of alcohol.

The behaviour with alcohol remained quite unchanged as the pressure was reduced below that of the atmosphere, in both air and carbonic acid. The cone form of meniscus was observed in air at a pressure of 12 cms. of mercury and in carbonic acid at a pressure of 25 cms. of mercury, these being the lowest pressures used with these gases. Moreover the potentials at which the current ceases were found to be the same in both gases and for all of the pressures used. This potential was, however, only about six-tenths of that at which the discharge stopped in air.

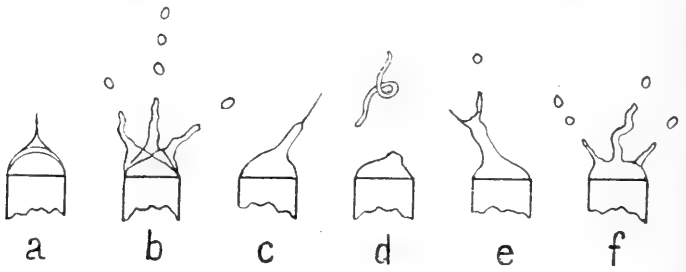


Fig. 3.

8. When the cone-shaped discharge surface of alcohol was examined with a microscope magnifying about 50 diameters in the instantaneous light of a Leyden jar spark, the dark streak seen along the prolonged axis of the cone was not resolved into drops. When examined without a microscope in weak continuous light, a form of brush of considerable angle was faintly seen extending outward from the end of the cone; but this brush showed no luminosity when examined in the dark. A luminous brush of the same form as this one was previously observed (§ 2) at the end of a hemispherical drop of water at each of the intermittent discharges from it.

9. When an alcohol meniscus is examined under instantaneous illumination as a current first starts to flow from it (i.e. during the intermittent stage), some of the shapes that the meniscus is seen to assume are indicated by "*a*" in Fig. 3. The peaked form tapers down to a point of vanishing diameter.

With voltages at which the surface shows a great deal of agitation when viewed in ordinary light, it is seen under instantaneous illumination that large masses and threads of liquid are pulled out of the surface. Some of the forms which have been observed are shown in Fig. 3 by the drawings "*b*," "*c*," "*d*," "*e*" and "*f*."

10. After the current from the conical form of discharge with alcohol has been allowed to flow for some time, the plate underneath the point becomes wet with a disc of liquid which has a diameter about equal to the distance between the point and the plate. From the amount of liquid thus carried over, it appeared possible that the whole current from the alcohol point might result from the charges carried by minute drops leaving the charged surface and impinging on the plate.

Experiments to test this point were made on the conical form of discharge in the open air, by noting what effect a blast of air, blown at right angles to the electric field between the point and plate, would have upon the current received by the plate. The receiving plate was divided into two parts, one of which was earthed directly and the other of which was earthed through a galvanometer. When the discharge point was placed directly over the division line between these two parts and a moderate blast of air from a foot-bellows was directed along the surface of the plate from the part connected to the galvanometer towards the second part, the current reaching the galvanometer was reduced to zero although the blast had no appreciable effect upon the discharging surface. When the point was moved so as to be over a portion of the plate at some distance from its edge, the same stream of air reduced the current flowing through the galvanometer but did not completely destroy it.

Under similar circumstances a current from a metal point, carried as it is by gaseous ions, would not have been appreciably affected by the stream of air which was used. It is evident, therefore, that all of the current in this case was carried by drops of liquid, much larger than ordinary ions.

A rough estimate of the size of these drops may be made from the observation that with a current of 2000 electrostatic units the conical point discharged alcohol at the rate of 10^{-4} cubic centimetres per second. The point had a radius of 0·0189 cm., was situated 1·3 cms. above the plate and was maintained at a potential of 3300 volts. The electric surface density on the drops must be

less than what would correspond to a drop having the potential of the discharge point, and so an estimate was made based on later considerations and it was found that the radius of the drops is probably between 10^{-5} cms. and 10^{-4} cms. Accordingly many millions of these drops must be formed every second under the conditions existing in the conical form of discharge from an alcohol surface.

The ejection of these comparatively small pieces from the surface is evidently quite a different matter from the outflow of large drops from the tube, and is doubtless the result of a state of instability of the surface.

Conditions for Instability.

11. The conditions that are necessary for a charged surface to become unstable are exemplified by considering the equilibrium equation for a spherical drop

$$\frac{2T}{r} - p - 2\pi\sigma^2 = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1),$$

where T is the surface tension, r the radius of curvature, σ the electric surface density, and p the excess pressure inside the drop over that outside.

The equilibrium becomes unstable when, p remaining unchanged, a small outward displacement of a portion of the surface increases the outward force due to σ, because of the increased curvature produced, more than it does the restoring pressure arising from surface tension.

To obtain a solution by this simple method, it is necessary to know not only the shape of the surface but also the distribution of the charge on it, both before and after the displacement in question.

The surface of a small drop at the end of a vertical tube is one of revolution and is nearly spherical. Let us consider the surface to be an ellipsoid of revolution about the axis of the tube, and consider a displacement arising from an elongation of the rotation axis. This elongation might result from liquid flowing into the drop from the tube above, the minor axis remaining unchanged, or it might result from a contraction of a portion of the drop near its place of attachment, the change of shape at the apex approximating more or less closely that of an ellipsoid of constant volume. The calculation of the distribution of the charge on any shaped drop suspended at the end of a cylinder is rather difficult. However, it is of interest to consider some ideal cases of isolated, charged, ellipsoidal drops, undergoing changes of shape as a whole in one of the two ways mentioned above, namely by keeping the minor axis

of the generating ellipse constant and hence having a variable volume, or by keeping the volume constant.

Let "c" be the semi-axis of rotation and "a" the transverse semi-axis, and consider in each case the equilibrium of a portion of the surface at the end of the axis of rotation, inasmuch as experiment showed the instability to begin at this place on the drops examined.

12. First, consider the volume of the ellipsoid constant and that it has a constant charge Q. Equation (1) becomes

$$\frac{2Tc}{a^2} - p - \frac{Q^2}{8\pi a^4} = 0 \quad \text{.....(2).}$$

Putting the first derivative equal to zero, to get the relation for the limit of stability, and remembering that $a^2c =$ a constant,

$$Q^2 = 16\pi a^2 cT \text{.....(3).}$$

Equation (2) shows that for this value of Q^2, the excess pressure, p, inside the drop vanishes.

For a sphere, equation (3) becomes

$$Q^2 = 16\pi a^3 T \text{.....(4),}$$

a result obtained long ago by Lord Rayleigh*.

If Q and "a" are taken as constant, there is no instability, but for every value of Q there is a value of "c" which gives stable equilibrium.

Since in the drops experimented upon the potential V, and not Q, was maintained constant, cases will next be considered with that supposition.

13. Consider V and "a" constant. Equation (1) becomes

$$\frac{2Tc}{a^2} - p - \frac{c^2e^2V^2}{2\pi a^4 \log^2 \dfrac{1+e}{1-e}} = 0 \quad \text{.....(5),}$$

the ellipsoid being considered prolate, and "e" being the eccentricity of the generating ellipse. The relation at the limit of stability is

$$V^2 = 2\pi aT \frac{\sqrt{1-e^2}\log^3 \dfrac{1+e}{1-e}}{\log \dfrac{1+e}{1-e} - 2e} \text{.....(6),}$$

which becomes for a sphere, at the limit $e = 0$,

$$V^2 = 24\pi aT \text{.....(7).}$$

* Lord Rayleigh, *Phil. Mag.*, Series 5, Vol. 14, p. 184, 1882.

14. Lastly, considering V and the volume of the ellipsoid constant, instability sets in when

$$V^2 = 4\pi a T \frac{\sqrt{1-e^2}\log^3\frac{1+e}{1-e}}{(1+e^2)\log\frac{1+e}{1-e} - 2e} \quad \ldots\ldots\ldots\ldots(8),$$

and this expression, at the limit of a sphere, reduces to

$$V^2 = 12\pi a T \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(9).$$

It is seen that under the conditions here taken, when instability begins, the pressure inside the drop is greater than that outside the drop, whereas under the conditions assumed in § 13 the opposite is the case.

In the experiments with hemispherical drops of water at the ends of tubes it was observed that when instability commenced there was an excess of pressure on the inside of the drops and this was approximately equal to one-tenth of the whole pressure caused by surface tension alone. With a low meniscus however, the pressure inside the drops, under the same conditions, was found to be less than that outside.

It may be said that the values of V^2 given by equations (6) and (8) differ very little from those given by equations (7) and (9) respectively, for values of e as high as 0·2 or more.

15. The preceding results, and some further considerations, lead to the general expression (applicable to drops on the end of a tube)

$$V^2 = CaT \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(10),$$

as giving the potential at which instability commences, C being a constant depending upon the shape of the drop and the manner in which it undergoes axial changes. We may test the expression experimentally by finding how the potential necessary for instability is dependent upon the size of the tube and upon the surface tension of the liquid used.

Experimental Results.

16. As the first example we will take some results obtained for water and alcohol. The potentials at which these surfaces ceased to be unstable were found to be 4050 volts and 2350 volts respectively. In agreement with our formula, the squares of these numbers are seen to be proportional to the surface tensions of the two liquids (taking 72 for the value of the surface tension of water and 24 for that of the alcohol used, which was nearly absolute).

As a further test of the surface tension factor in formula 10,

results, obtained some time ago with a number of liquids, are given in Table I for the potentials at which the surface oscillations, indicative of instability, ceased as the voltage was decreased. The list of liquids is confined to those which have an appreciable conductivity and still do not leave a solid residue on evaporation. The experiments were done in the open air (pressure = 74 cms. of mercury) with the point at a distance of 1·5 cms. from a plate.

TABLE I. Instability Potentials.

Radius of point, 0·0250 cm. Average temperature, 23° C.

Substance	Density	Surface tension	Instability potential	C
Acetone	0·790	24·9 dynes/cm.	9·30 electrostatic units	140
Methyl alcohol	0·804	25·3 ,,	9·25 ,,	137
Ethyl alcohol ...	0·812	25·3 ,,	9·25 ,,	137
Water	0·998	72·0 ,,	15·90 ,,	140
Acetic acid	1·053	30·0 ,,	9·67 ,,	125
Glycerine	1·259	65·2 ,,	15·17 ,,	142
Chloroform	1·483	27·0 ,,	10·17 ,,	153

Column 2 gives the densities of the liquids named in column 1. Column 3 gives the values of the surface tensions of the liquids, determined while in the apparatus by measuring the pressure in an uncharged hemispherical drop. These values are not very accurate as the readings for them were taken incidentally only, as the experiments were made for another purpose for which the surface tensions were not needed. The value given for the surface tension of glycerine was taken from tables, as a difficulty was found in the direct measurement owing to its high viscosity; and the value for methyl alcohol is taken the same as that of ethyl alcohol since no direct measurement was made and the two substances have very nearly the same value.

Column 4 gives the observed instability potentials expressed in electrostatic units. The last column gives the values of the constant C in expression (10), as calculated for each liquid from the tabulated values of V, a, and T.

When account is taken of the rather large experimental errors in the determinations of T, the values of C are nearly enough alike to show that the squares of the potentials at which instability begins are proportional to the surface tensions of the liquids.

17. For showing the dependence of V upon the radius of the discharge point, reference is made to a table of electric intensities, f, given in a previous paper (*loc. cit.*, p. 88) which were determined at the surfaces of water drops ranging in radius from 0·0146 cm.

to 0·0543 cm., when the oscillations in question and the currents from the points were observed to stop. Since the current from a point also starts at this stage it was then thought that the values of f obtained were those requisite for starting a discharge, but it is known from the considerations given in this paper that the oscillations indicate rather the beginning of instability at the surface, although with water the two actions start at potentials which are not greatly different.

The results, in the table referred to, show that $f\sqrt{a} =$ a constant, and putting $f \propto \frac{V}{a}$, we get $V^2 = a \times$ a constant, which is the relation in expression (10).

Taking the average value of the constant C from Table I, expression (10) becomes $V^2 = 140aT$ and for the sake of comparison with the expressions obtained in §§ 13 and 14, this may be put in the form $V^2 = \frac{16\pi aT}{0{\cdot}36}$.

It was found experimentally (*loc. cit.*, p. 74) that the surface density on the end of a hemispherical drop attached to the end of a tube, with a plate at a distance of 1·5 cms., is only about six-tenths of that on an isolated sphere of the same diameter and having the same potential. The square of this number, or 0·36, must be introduced into the denominators of the theoretical expressions in §§ 13 and 14 to permit of their comparison with experimental results on drops at the ends of tubes. The experimental coefficient 16 is seen to be between the two coefficients 24 and 12 obtained in §§ 13 and 14 respectively, and this may be taken to indicate that the experimental drop changes form in a way intermediate between that of keeping a constant volume and that of keeping a constant cross-section.

Discussion.

18. The relation of the potential at which the surface of a charged drop becomes unstable, to that at which an electrical discharge starts, will now be considered. A study of a table giving surface tensions indicates that in air at atmospheric pressure the limit for instability is reached on the surface of drops of all substances which are liquid at room temperature, with the exception of mercury, before a potential is reached at which an electrical discharge through the air begins. But in hydrogen for example the potential for an electric discharge from a water surface is smaller than the potential for the beginning of instability.

The size of the tube on which the drop is formed is of no great consequence for the relationship in question, since it has been shown*

* J. Zeleny, *Physical Review*, Vol. 25, p. 317, 1907.

that for metal points with a radius greater than 0·025 cm. the voltages at which a discharge begins are proportional to the square roots of the radii of the points, which is the same relation as that obtained above for the beginning of instability with liquids, although for points smaller than the value given the voltages necessary for starting the current are larger than this relation would indicate.

After a discharge has started from a surface, a large increase in the voltage may not bring on instability, because while a current is flowing the electric intensity at the surface varies but little with the voltage used (*loc. cit.*, p. 85).

Since the potential necessary for a discharge depends upon the pressure of the gas while the potential for the beginning of instability does not, it is possible to have instability begin first in a gas at atmospheric pressure and to have a discharge begin first in the same gas at reduced pressures.

19. With water surfaces in air at atmospheric pressure, the potential at which a discharge starts is only a few per cent. higher than the potential at which instability sets in, and as the potential is gradually raised above this latter value the instability at the surface stops suddenly and a steady current flows from the surface. This indicates that either the surface tension is increased by the current or the electric intensity at the surface falls slightly as soon as the current starts. There is some evidence for both views.

A sort of electrical discharge is present with water surfaces as soon as instability begins as is shown by a faint luminosity seen extending some distance from the point, but apparently this discharge is brought on by the changes of shape caused by the instability and may be confined altogether to the surfaces of the flying drops.

The conical form of discharge surface, seen when carbonic acid is used with a water surface or when liquids of low surface tension are used, makes its appearance over a certain range of voltages in cases where the instability voltage is considerably lower than the discharge voltage. As the liquid is being pulled out rapidly from the tip end of the meniscus in the form of threads, the surface distribution over the rest of the meniscus becomes altered so that equilibrium necessitates the conical form of surface. This form of surface should be obtainable also with a water surface in air, if the discharge potential were raised sufficiently. The latter condition can be produced by increasing the pressure of the air, and on trial it was found that the conical form came on as predicted when the pressure of the air was increased but 10 cms. of mercury above that of the atmosphere.

The presence of a little alcohol vapour in the discharge vessel also made the conical form of surface appear, by lowering the surface tension of the water.

20. The facts noted in §§ 5 and 7 that with water in carbonic acid and with alcohol in air or carbonic acid, the oscillations of the meniscus began at a potential in each of the cases which was independent of the pressure of the gas over a considerable range, are explained by the discharge potential being much higher at atmospheric pressure than the instability potential, so that even at much lower pressures the discharge voltage was not reduced enough to be below the potential for instability.

The successive individual discharges, mentioned in § 2, which are observed at increasing potentials when a discharge is first started from an old water surface, are doubtless caused by the surface having a low surface tension owing to contamination. As pieces of the liquid are pulled away, the contamination gradually disappears and a higher and higher voltage is necessary to produce instability. The falling of the meniscus after each of these discharges shows that an increase in the surface tension has taken place.

At a later stage where the surface undergoes continuous oscillations at a given voltage, a different cause doubtless enters, the surface becoming stable intermittently owing to electric shielding by the material pulled out of its surface.

Sporadic discharges like those considered above are often noticed with metal points at voltages far below the final discharge potential, and it is possible that something of the same nature takes place there as in these cases, a part of the adsorbed layer of gas and water being pulled off from the surface at each discharge.

21. The expression given in § 15 for the potential at which instability begins applies to cases where the medium between the point and plane is a gas, and thus having a dielectric constant of approximately unity. If the experiment were done in a non-conducting fluid not miscible with the liquid of the drop, the expression would have to include k, the dielectric constant of the new medium, and would be written $kV^2 = CaT$. Evidently, the dielectric constant could be determined in this way by measuring the surface tension between the two liquids and the voltage at which instability starts.

22. It follows from the results recorded in this paper that before a potential is reached which would result in lightning striking a tree, for example, the leaves, if wet, will discharge water from their edges in the form of minute electrified drops which will form an upward shower tending to neutralize the electric field. But since the discharge potential is so near to the instability potential for water, it is not likely that this phenomenon plays any important part in the atmospheric electricity during storms.

The condition for instability also sets a limit to the possible size of charged rain drops, but the charge on drops of the usual size would have to be very large indeed before instability would arise from this cause.

Summary.

23. Certain motions of agitation, which are observed at the surfaces of drops of liquid when an electric current starts to flow from them, are described and the conditions under which the movements occur are studied (§§ 2—9).

It is shown that in some cases (§ 10) the observed current is all carried by minute charged drops, formed of liquid which has been pulled from the discharging surface.

The ejection of these masses from the surface occurs when the stabilizing effect of surface tension is overcome by the instability caused by the electric forces (§ 11).

A relation between the potential at which instability begins, and the dimensions of the drop and the surface tension of the liquid, is derived for some ideal cases (§§ 11—14), and a formula applicable to the drops used in the experiments is given (§ 15).

In agreement with theory, the experiments show that the square of the potential at which instability begins is proportional to the surface tension of the liquid (§ 16) and to the radius of the tube on which the drop is formed (§ 17).

In air at atmospheric pressure, instability sets in before the true electric discharge through the gas begins, for all liquids except mercury (room temperature assumed) (§ 18).

In a gas like hydrogen or in air at reduced pressures, the electric discharge may be obtained in many cases before instability occurs (§ 18).

An explanation of the observed effects is given (§§ 19, 20), and some outside applications of the results obtained are considered (§§ 21, 22).

A part of the work on this investigation was done at the Cavendish Laboratory, and I wish to express here my thanks to Prof. Sir J. J. Thomson for his kind and helpful interest in the work and for placing the necessary apparatus at my disposal.

PROCEEDINGS

OF THE

Cambridge Philosophical Society.

The Origin of the 'Wolf-note' in Bowed Stringed Instruments. By G. W. White (M.Sc., 1851 Exhibition Scholar), Fitzwilliam Hall. (Communicated by Professor Sir J. J. Thomson.)

(Plates II—III.)

[*Read* 8 February 1915.]

On all stringed instruments of the violin type a certain pitch can be found which it is difficult and often impossible to produce by bowing. This note is called the 'wolf-note' and usually occurs at an interval of an eleventh or twelfth above the lowest note of the instrument. At this pitch on either string the bow refuses to 'bite,' and a soft pure tone is almost impossible to obtain; if the pressure of the bow on the string is increased the tone resulting is usually of an unsteady nature with considerable fluctuations in intensity (the note probably received its name from this phenomenon). It was thought that possibly an analysis of the vibration of the belly of the instrument for frequencies in the neighbourhood of the 'wolf-note' would settle the question of its origin, on which there is considerable disagreement amongst musicians. The instrument selected for experiment was the 'cello. On such instruments the imperfect note is usually very pronounced especially when played high up on the G-string.

The vibration of any part of the body of the instrument was recorded as a line on a moving photographic plate by reflecting a beam of light from an optical lever one leg of which rested on the vibrating surface. Fig. 1 shows the method of holding the 'cello. A rigid wooden framework was clamped tightly to a table. The framework served to hold a steel rod in such a position that it stood parallel to and at the same height as the neck of the instrument when standing vertically with its peg on the table.

By means of adjustable clamps attached to the rod the instrument could be securely fixed. At the spot on the belly chosen for investigation a strip of wood covered with a piece of microscope cover glass was fixed. This formed a support for one leg of the optical lever. The other two legs rested in a conical hole and V-groove made in a bracket attached to the edges of the belly and back (the edges joined by the ribs are practically free from vibration). The optical lever was kept with its legs horizontal and in contact with the belly and the bracket plate by means of two rubber bands. The mirror of the optical lever was attached directly to the plate carrying the legs and therefore rocked about a vertical axis. A beam of light from a pin-hole illuminated by an arc lamp was directed on the small concave mirror and focused on the photographic plate. Half plates were used and were fixed in a dark slide which could drop in a framework. The latter could itself be moved horizontally thus allowing ten or more exposures to be made on one plate. The dark slide was held at the top of the rails by an electromagnet, and at the required moment was dropped by breaking the circuit with a lever under the experimenter's foot. The optical lever was placed with its movable foot on the C-string side of the belly and near the F-hole.

First the author satisfied himself that the main features of a vibration curve however complex could invariably be reproduced on repeating the note. Curves 1—3 (Fig. 2) were obtained by bowing the open D-string with different intensities. A slight difference in the prominence of partials will be observed.

Secondly it was ascertained how the character of the vibration curve for a given pitch depended on the point examined on the belly. Three points on the belly were studied and such small differences as were obtained in the vibration curves could be completely accounted for by the changes in amplitude. Curves 4—6 (Fig. 2) were obtained by bowing A on the G-string and recording the vibration of a point on the belly (*a*) near the left foot of the bridge, (*b*) 9 cms. above the top of the left F-hole and (*c*) 9 cms. below the bottom of the left F-hole. The magnification was arranged to suit the amplitude obtained.

It was also observed that the character of the belly vibration produced by bowing a string was often greatly affected by the forced vibrations of the other three strings if these were free to vibrate. Curve 7 (Fig. 2) was obtained by bowing the open G-string, the other three strings being free; the belly vibration changed to that shown in curve 8 on fixing these strings. In the following examination of the vibrations produced by bowing a string other strings were always fixed.

Feeling satisfied from the above and similar tests that the

photographs could be relied upon, the author proceeded to obtain records for a series of notes played on the G-string and passing through the 'wolf-note.' The curves shown in Fig. 3 were obtained for such a series with a 'cello which had a 'wolf-note' at a pitch a little below G. The notes in the series start at B below the 'wolf-note' and proceed by two steps per semi-tone through the 'wolf-note' to A. In the curves the quarter tones are indicated by *plus* signs. Two records of the 'wolf-note' were made.

It will be observed from these photographs that for notes played low down on the G-string the curve is rather complex, the third and fourth partials being quite prominent. As the note approaches the 'wolf-note' the curve becomes exceedingly simple and *a considerable increase in amplitude occurs.* It will also be noticed that the character of the 'wolf-note' curve changes along its length, unlike other curves. This was shown still more clearly in photographs which were taken on a film wound on a drum of diameter 20 cms., which while rotating moved horizontally on a screw of pitch 8 mms. Fig. 4 is a section taken from a film record of this nature. The curve clearly shows the phenomenon of 'beats,' which explain the fluctuations in intensity of the sound.

The increased amplitude of the belly at the 'wolf-note' led one to the conclusion that this note, or some note near it, was the pitch of best resonance of the instrument. [The author satisfied himself that the effect was not one due to a change in pressure of the bow, by studying the connection between the amplitude of the curve and the bow pressure for pitches in the neighbourhood of the 'wolf-note.'] To find the 'resonance note' a cornet was blown with its bell near the A-string corner of the bridge of the 'cello, all strings of which were fixed, and the motion imparted to the optical lever examined. Any frequency could be obtained on the cornet by adjustment of the valves and slides. The maximum belly vibration occurred when a note between F-sharp and G was sounded. On the 'cello examined the 'wolf-note' could be heard between these two pitches; as the note played was raised in pitch from F, the pure tone ceased at F-sharp and was resumed at G. The pitch of the 'wolf-note' was therefore the pitch of maximum resonance of the instrument. Another 'cello was examined which had its 'wolf-note' at F (a little lower than in the previous case). This instrument also was found to respond best to a cornet note of the same pitch as the 'wolf-note.'

From the above results it is clear that the 'wolf-note' does not arise from imperfect workmanship, as is commonly thought, but is a defect natural to the instrument when made according to the present dimensions; these have been found empirically to give a good general tone, and incidentally they give the instrument a maximum resonance pitch near the middle of its range. Playing

a bowed stringed instrument consists in forcing a system to vibrate with certain frequencies, and the 'wolf-note' arises when the impressed pitch approaches the 'natural pitch' of the system. The fluctuations in intensity are due to beats which accompany the forced vibration impressed on the resonator and which by reacting on the string may interfere with even bowing.

These preliminary experiments were carried out in the University of Bristol at the suggestion of Dr A. M. Tyndall to whom I wish to express my best thanks. A complete study of the problem is now being made which will deal with the nature of the resonator, the mechanism producing the imperfect quality and methods adopted to minimise the effect of the 'wolf-note.' The results of this work will be published shortly in a joint paper with Dr Tyndall.

My thanks are also due to Dr H. Brown, F.R.S., for his interest in the problem and his kindness in placing an instrument at my disposal for the experiments.

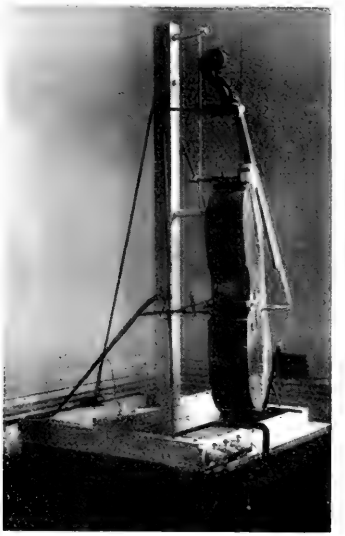


Fig. 1.

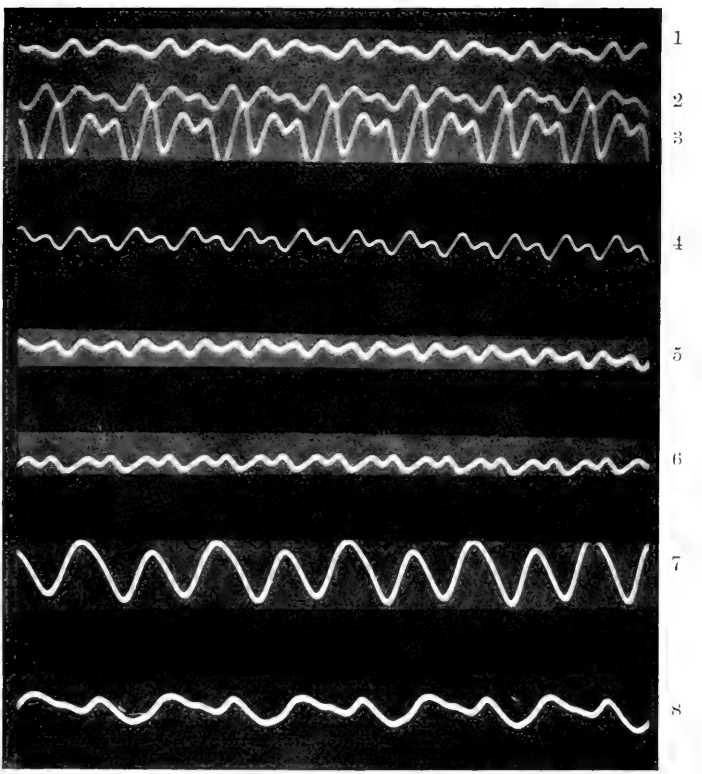


Fig. 2.

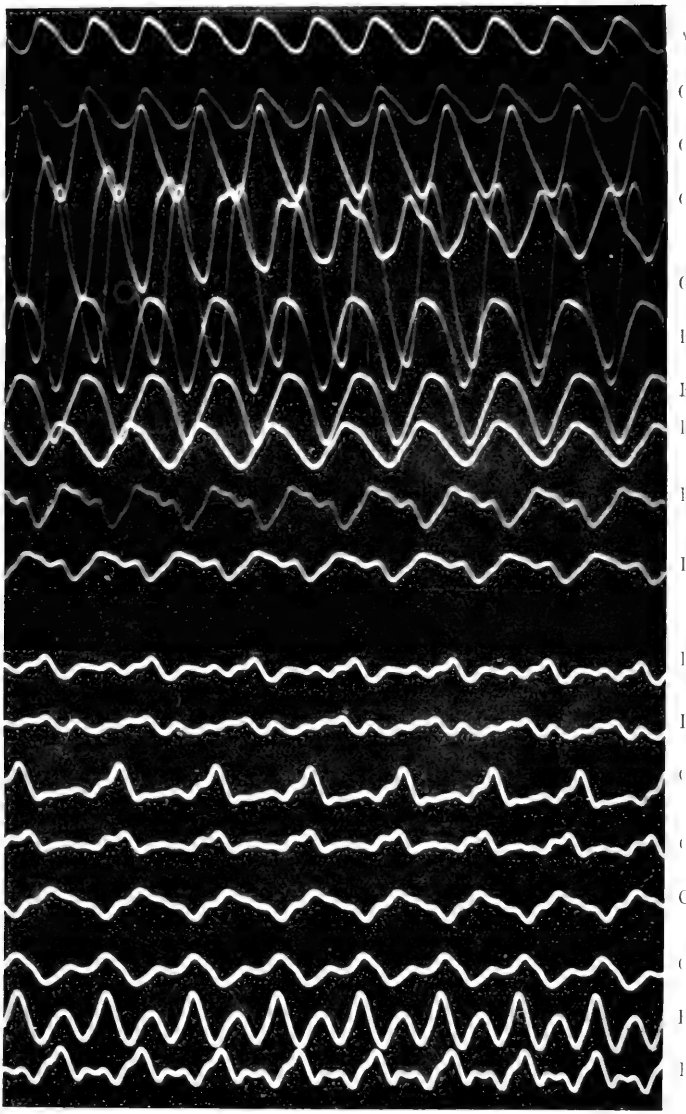


Fig. 3.

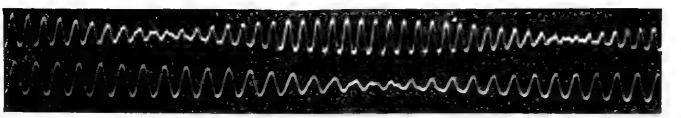


Fig. 4.

On some fossil plants from the Devonian rocks of North Devon. By E. A. Newell Arber, M.A., Sc.D., F.G.S., Trinity College, and R. H. Goode, B.A., St John's College.

(Plates IV—V.)

[*Read* 22 February 1915.]

Introduction.

Before the completion of the work on the Carboniferous rocks of Devon and Cornwall of the senior author of this note, a search had been begun for plant remains in the Upper Devonian series of North Devon. The existence of a few fossil plants, collected by the late Townsend Hall from these beds, seemed to indicate that further work might bring to light an interesting Devonian flora. The results however of this inquiry, which was aided by a grant from the Royal Society Government Grant Committee, have been disappointing. The repeated visits of the author in question spread over a considerable number of years, and supplemented by the enthusiastic help and skilful collecting of Mr D. G. Lillie, M.A., of Cambridge, Dr Young, of Woolacombe, and Mr I. Rogers, of Bideford, have not resulted in the discovery of fresh specimens at all proportionate to the time, labour and expense involved.

There is little doubt that, while fairly well preserved plants do sometimes occur at more than one locality situated on the Baggy or *Cucullæa* beds, and to a less degree in certain other divisions of the Devonian of North Devon, such plant-bearing sediments are always of the nature of very thin local lenticles. These impersistent layers, often of less than an inch in thickness, are only occasionally exposed in quarries, and thus the chances of finding them are small. The well-known Sloly Quarry, in the parish of Marwood, some three miles north of Barnstaple, is a case in point. Many years ago it would appear that a plant-bearing band was there exposed, and of this exceptional opportunity Townsend Hall made good use. Nearly all the Devonian specimens from Devon, which now find a home in our museums, were collected by him in these circumstances. This quarry has recently been repeatedly visited and searched by one of us, and it seems quite certain that no plant-bearing shales are to be found there at present. In fact, beyond the occurrence of some very fragmentary and badly-preserved stem-casts in sandstone, of no value, no plant remains have been obtained from it for many years past.

Again, in the case of the small quarry of Croyde Hoe, on Baggy Point, from which some of the specimens figured here were

collected, it has been found that the plant-bearing shales are less than an inch in thickness, and very impersistent. Considerable quarrying operations, involving blasting, were found necessary to obtain even the few fragments figured here.

Previous Records.

Fossil plants thus appear to be exceedingly rare in the Devonian rocks of North Devon. Yet the records of such are more numerous than might be supposed. There is however reason to believe that many of these relate to very obscure forms, no doubt originally of vegetable origin, but now quite unworthy of even generic determination. Some may also be the tracks of animals, others structures of inorganic origin.

The earliest reference dates from 1838, when Williams* mentioned the "Wollacombe sandstones and purple slates—containing fossil wood and plants," and also "the wood and plants of the Sherwell sandstones."

Two years later Sedgwick and Murchison† published Lindley's determinations of plants, collected from Sloly quarries (Marwood beds) by Williams and also by Major Harding‡. Lindley concluded that "these remains are not susceptible of *specific* identification." A *Stigmaria* or a *Lepidodendron* were recognised however, and other specimens were doubtfully compared with *Sternbergia* and *Calamites*. Lindley§ also identified *Stigmaria ficoides* from the Marwood quarries, and, according to De la Beche‖, his other determinations were a "*Bothrodendron*, somewhat resembling *B. punctatum*, and a *Knorria*, like *Kn. Sellonii* (Sternberg)."

In 1863 Salter¶ recorded from the Marwood beds "*Bornia* (*Calamites*) *transitionis*, Gœppert and *Lepidodendron* (*Knorria*) *dichotomum* Haughton, and its roots."

In 1866 Godwin-Austen** mentioned specimens from the Foreland Grits, which he regarded as due originally to a "terrestrial vegetation." Etheridge†† however dismissed them as "a few undeterminable plant-like remains (Fucoids)."

* Williams, *Rep. Brit. Ass. Liverpool* (1837), Vol. vi. pt. 2, p. 95, 1838.

† Sedgwick and Murchison, *Trans. Geol. Soc.* Ser. 2, Vol. 5, pt. 3, pp. 648, 682 and 695, 1840.

‡ This collection, which is now in the Jermyn Street Museum (Nos. 14881–3), consists solely of *Knorria* casts.

§ Sedgwick and Murchison, *ibid.* p. 690.

‖ De la Beche, *Rep. Geol. Cornwall, Devon and W. Somerset*, London, p. 50, 1839.

¶ Salter, *Quart. Journ. Geol. Soc.* Vol. 19, p. 480 (see also p. 481), 1863.

** Godwin-Austen, *Quart. Journ. Geol. Soc.* Vol. 22, p. 3, 1866.

†† Etheridge, *Quart. Journ. Geol. Soc.* Vol. 23, p. 595, 1867; see also *Proc. Geol. Assoc.* Vol. xxv. p. 100, 1914.

In the following year the most important paper which has yet been published on the Devonian plant fossils of Devon appeared from the pen of Townsend Hall*. He recorded *Sigillaria* sp. from Top Orchard quarry, *Calamites* sp. from Frankmarsh quarry, and *Sternbergia* sp. from Croyde,—all from the Pilton beds. We have not been able to trace these specimens. There is one from the locality first mentioned above in the Atheneum, Barnstaple, but if this is the specimen in question, it is quite indeterminable.

In addition to previous records, the following important determinations were made from Sloly quarry:

Adiantites hibernicus Forbes,
Knorria sp.,
Sphenopteris sp.

Knorria sp. is also recorded from Marwood, but no plants were known to Townsend Hall from Baggy Point, with the exception of "doubtful species."

Etheridge†, in his important memoir on the Devonian of North Devon, enumerates the same determinations, and records *Adiantites hibernicus* Forbes from both the Baggy and Pilton beds.

Townsend Hall's collection appears to be preserved partly in the British Museum (Nat. Hist.) and partly in the museum of the Atheneum, Barnstaple. One of us has carefully examined these fossils more than once, but has been unable to find any specimen of *Archæopteris hibernica* (Forb.) among them. We may add that we have never seen any example of this plant which there was any reason to suspect of having been derived from Devonshire rocks. Thus this record is best regarded as "not proven."

In 1896 Whidborne‡ recorded *Knorria* sp., *Bernia* [*?Bornia*] sp., *Rhodea moravica* ? and Stigmarian roots from the Marwood, Baggy and Sloly beds. It is possible that the attribution to *Rhodea* relates to the fossils here referred to *Sphenopteridium rigidum* (Ludw.).

Hicks§ also mentions plant remains in the Hangman Grits at Timberscombe, West Somerset, and from the Pickwell Down Sandstones, near Barlinch Abbey, in the Exe Valley. These specimens however we have been unable to trace.

One of the most recent lists‖ of plants from Devonshire, some

* Hall, *Quart. Journ. Geol. Soc.* Vol. 23, p. 380, 1867.
† Etheridge, *ibid.* pp. 616, 669, 674.
‡ Whidborne, *Proc. Geol. Assoc.* Vol. xiv. p. 372, 1896.
§ Hicks, *Quart. Journ. Geol. Soc.* Vol. 53, p. 440, 1897.
‖ Kidston, in Woodward *Geol. Mag.* Dec. 3, Vol. 1, footnote on p. 534, 1884; also in Woodward "Monogr. British Carbonifer. Trilobites," *Pal. Soc.* footnote p. 59, 1884.

of which may have been derived from Devonian, while others were certainly obtained from the Carboniferous rocks, must be discarded, as there were no records as to the localities from which any of the specimens were collected. The circumstances relating to these records have already been explained by one of us* elsewhere.

To sum up the results of previous work on the Devonian plants of North Devon, we may conclude that the only valid and satisfactory determinations so far all relate to specimens from the Baggy beds, and that they consist of *Sphenopteris* sp., the *Knorria* casts of some Lycopodiaceous genus, and very doubtfully *Asterocalamites scrobiculatus* (Schl.). The best examples of these will be described here, with an account of some specimens from a new locality at Baggy Point, which was originally discovered by Dr Young in 1907. We may commence with the first plant record from the Lynton beds, from material collected by Mr G. F. Tregelles, of Barnstaple, in 1911.

Description of the Specimens.

An obscure Plant impression from the Lynton Beds.

Text-fig. 1.

Text-fig. 1 represents one of the plant remains† obtained by Mr Tregelles from a quarry to the west of Lynton at the head of the Valley of the Rocks, in a lane behind "Rock Lodge." The mineral representing the plant somewhat recalls graphite in appearance. It consists of a short axis bearing several lateral organs, which, despite the absence of any nervation, are more leaf-like than branch-like in form. The axis is fairly stout, incomplete, especially above, and measures at least 3·8 cm. in length and 4·5 mm. in width at the base. The lateral organs appear to be arranged in a close spiral, there being eight or more, all in continuity. The bases of several others are also seen broken off abruptly. None however are complete. They measure from 7 to 10 cm. in length. They are lanceolate in form, and the apex appears to be elongately acute. They are 7—8 mm. across at their broadest part. They show no trace of nerves, and there is no real evidence of any dichotomy. The division of one of the lateral members, which is seen on the left-hand side of the specimen (text-fig. 1), is in all probability due to bad preservation, and not a natural feature.

* Arber, *Phil. Trans. Roy. Soc.* Ser. B, Vol. 197, p. 298, 1904.

† We understand that another similar specimen has been presented by Mr Tregelles to the Atheneum Museum, Barnstaple. The figured specimen was given by him to the Sedgwick Museum, Cambridge.

This impression, although admittedly obscure, appears to us to be of a new type, and quite distinct from those generally classed under the term *Bythotrephis* Hall, some of which are undoubtedly of algal origin, while others may in reality be the tracks of animals. It is distinguished especially by the absence of any dichotomy of the lateral organs from such plants as *Psilophyton* Daws. and *Hostimella* Stur. The nearest approach in habit among Devonian fossils previously described appears to be a specimen from Spitzbergen figured by Nathorst* in 1894 as a *Psilophyton*-like stem, and one from Canada attributed by Dawson† to the same genus. Here we also find an axis bearing lateral organs, somewhat similar in form to our specimen, but they are apparently arranged distichously, and not spirally. In

Fig. 1. An obscure plant impression from the Lynton beds. (Reduced.)

any case, these fossils are probably generically distinct from our specimen.

At the same time, the British plant appears to us to be too obscure to warrant the institution of a new generic term. Its chief interest lies in the unusual habit, which, imperfectly preserved as it is, inclines us to the view that we may here be dealing not with a terrestrial, but with an aquatic plant, though not necessarily a marine organism. However this may be, the plant origin of the specimen will, we think, hardly be disputed, and it is, as has already been pointed out, the first example made known from the Lynton beds.

* Nathorst, *K. Svensk. Vetens. Akad. Handl.* Vol. 26, No. 4, 1894, p. 11, pl. I. fig. 1.

† Dawson, *Foss. Plants Devon. and U. Silur. Canada*, 1871, pl. VII. fig. 80*a*.

Upper Devonian Plants.

Sphenopteridium rigidum (Ludw.).

Plate IV, fig. 12; Plate V, figs. 1, 3, 6.

1869. *Sphenopteris rigida* Ludwig, Palæontogr. Vol. xvii, pt. 3, p. 117, pl. xxii. figs. 1, 1*a*, 1*b*.

1897. *Rhodea Schimperi* Potonié (pars), Lehrb. Pflanzenpal. p. 135.

1899. *Sphenopteridium rigidum* Potonié, *ibid.* p. 364, fig. 344.

1901. *Sphenopteridium rigidum* Potonié, Abhand. K. Preuss. Geol. Landesanst. N. F. Heft. 36, p. 16, fig. 2 on p. 18.

In the photograph reproduced on Plate V, fig. 1, which is slightly below natural size, several pinnules are seen, three of which are fairly complete. One is still attached to a broad axis, 4·5 mm. across. This specimen, which is in the Hall collection in the British Museum (Nat. Hist.) V. 3562, was derived from Sloly Quarry, and is the most perfect example of this frond yet obtained from Devonshire. It appears to be identical with *Sphenopteridium rigidum* (Ludw.) from the Devonian of Germany. As compared with Ludwig's original figures (see above) the agreement is perhaps not very close. The pinnules, which are 2 cm. in length, are more widely separated, somewhat less lobed and toothed, and the ultimate segments less pointed. Potonié's figures of the same specimens, especially fig. 2*c* (see above), appear to agree better, though here again the ultimate lobes or teeth are longer and more pointed. On the whole, however, we think there is little doubt as to the specific identity of the English and German fossils.

Several examples of the same plant have also been obtained by one of us from Baggy Point. They are all however more fragmentary than the British Museum specimen. One of them is figured on Plate V, fig. 6 (three times enlarged) showing some of the incomplete lobes of a pinnule. The terminations of a pinnule are also seen on fig. 3 of the same plate. In only one of our specimens is there any indication of the nervation (Plate IV, fig. 12). The nerves in each lobe are fairly close, several in number, and branched repeatedly.

These specimens we should also be inclined to attribute to *Sphenopteridium rigidum* (Ludw.). There is also some resemblance to *Sphenopteris devonica* Unger* from the Devonian of Saalfeld, Thuringia, though this plant is probably a distinct species.

* Unger, *Denksch. K. Akad. Wissen. Wien* (Math.-Nat. Cl.), Vol. xi. 1856, p. 163, pl. vi. fig. 21.

Sphenopteris sp.

Plate V, figs. 2 and 4.

Two small fragments of a further frond have been obtained from the Baggy beds of Croyde Hoe quarry, associated with *Xenotheca*. These are figured on Plate V, figs. 2 and 4. Fig. 4 shows two fragments of pinnæ, one with two, and the other with one pinnule. The pinnules are more or less orbicular in shape, with several rounded lobes, which are either entire or notched. They are contracted at the base. The nervation is not clearly seen, but appears to be of the *Sphenopteris* type. The axis is faintly striated longitudinally, but not winged.

Another specimen, shown on fig. 2 of the same plate, which we think may possibly belong to the same plant, has two undivided orbicular pinnules, the nervation of which is also obscure.

In general habit these specimens somewhat recall the common Coal Measure frond *Sphenopteris obtusiloba* Brongn., but the pinnules in the Devonshire specimen are much smaller, and the two plants are undoubtedly specifically distinct.

The resemblance is closer to the fragmentary and unique *Cyclopteris trifoliata*, Unger*, from the Devonian of Saalfeld in Thuringia, which for some, not very obvious, reason Schimper† places together with *Cyclopteris dissecta* in his genus *Triphyllopteris* as *T. elegans* (Ung.).

Crépin‡ has also figured another fragment from the Devonian rocks of Condroz (Belgium) under the name last mentioned above, and this again appears to be very similar to the specimen figured on Plate V, fig. 4.

The same is also true as regards the Lower Carboniferous type described by Stur§ as *Sphenopteris foliolata*, a much larger form with rounded pinnules and lobes.

On the whole, although the Devonshire specimens are certainly too fragmentary to warrant specific comparison, we should be inclined to regard them as very nearly allied to, if not identical with, the Devonian types described by Unger and Crépin, as mentioned above.

* Unger, *Denksch. K. Akad. Wissen. Wien* (Math.-Nat. Cl.), Vol. xi. 1856, p. 161, pl. vi. fig. 2.

† Schimper, *Traité Pal. Végét.* Vol. i. 1869, pp. 480—1.

‡ Crépin, *Bull. Acad. Roy. Belgique*, Ser. 2, Vol. 38, p. 362, pl. ii. fig. 6, 1874.

§ Stur, *Culm Flora*, Part 1, 1875, p. 22, pl. v. figs. 3—6.

Xenotheca devonica gen. et spec. nova.

Plate IV, figs. 1—7, 10—11. Text-fig. 2.

The most interesting fossil obtained from Baggy Point appears to be a cupule-like structure, or at least an organ connected with a fructification. A large number of specimens have been collected, but they are all fragmentary and in many cases badly preserved. They however appear to represent a quite new type, which is certainly of interest. A general idea of this fossil is given by the diagrammatic restoration shown on text-fig. 2. This, while not accurate in detail, is substantially correct.

Xenotheca, as we propose to term this new genus, consists of a dichotomously branched axis, forking several times, the finer branches being terminated by fairly large, cupule-like structures, which are here termed *thecæ*.

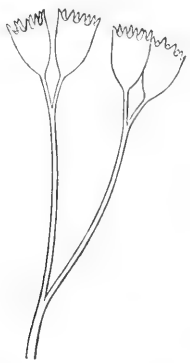


Fig. 2. *Xenotheca devonica* gen. et spec. nova. Restoration, about natural size.

Habit specimens. The general habit is that shown on fig. 10, Plate IV and text-fig. 2. The axis repeatedly forks, and each branch may end in a cup-shaped organ. No example has been obtained in which four *thecæ* undoubtedly occur at the ends of all four branches in continuity, but in the specimen figured on Plate IV, fig. 10 at least three and possibly four *thecæ* are present. In those examples in which the axis is least fragmentary the *thecæ* are usually badly preserved, and in this specimen it is impossible to decide whether one or two cups occur on the right-hand side. The length of the branches of the first dichotomy here is 2·2 cm., and of the second about 5 mm. The axis varies in breadth from ·5—1 mm.

In another example the length between the first and second dichotomies is 3·2 cm.

The Thecæ. The thecæ occur in several different states of preservation. One of the best of these is seen on fig. 11, natural size, and enlarged on fig. 7 of Plate IV. It is attached to a long stalk or axis. The theca is cup-shaped, rather short and comparatively broad. It measures about 8 mm. in length, and 1 cm. across at its widest part. Distally it is divided into at least eight lanceolate teeth, which are rather blunt. The teeth are about 1·5 mm. long. Another specimen (fig. 6 of the same plate) also attached to a stalk 1·4 cm. in length, and showing some indications of a dichotomy below, is somewhat similar but broader. It exceeds 1 cm. in length, and 11 mm. in breadth. The number of teeth is here uncertain, but they were at least eight, the points of which are broken off in nearly all cases.

A curious specimen, of which we have more than one example, is seen, natural size, on fig. 2 and three times enlarged on fig. 1 of Plate IV. It shows the ultimate dichotomy of an axis, each branch (7—8 mm. long) ending in a theca, the form of which is obscure. The number of teeth is also uncertain. They were at least four in each case, probably more. The thecæ, as well as the axis, are covered with a large number of minute oval or circular scars or prints. These consist of a small central elevation, then a broad circular depression, and finally an outer raised rim. A microscopic examination of these scars (two of which are shown 8 times enlarged on fig. 4) does not throw any more light on their origin and nature than can be ascertained by means of a hand-lens, and we are quite unable to explain them.

The theca shown on fig. 5 of the same plate is of a somewhat different type. Only a small part of the axis is seen, and the theca is more sac-like in form and the teeth are longer. The entire theca is 1·1 cm. long, and 5·5 mm. across at its broadest part. The teeth appear to be at least 5 mm. in length, and were probably more than six in number. This form is much narrower than in the case of those described above. It may be a distinct species, but on the whole we are inclined to include it, at least provisionally, under *X. devonica.*

Finally we have some examples of apparently lobed thecæ such as that seen on fig. 3 of Plate IV, where the divisions between the teeth extend nearly to the base in some cases, and only about half-way in others. How far this is to be regarded as a natural feature it is difficult to guess. On the whole we should be inclined to view such variations as at least partly due to accidents of preservation. It is conceivable of course that, when mature, the thecæ might split open along more than one line, but the present material does not allow us to form any opinion as to whether this actually did or did not happen naturally.

As to the real morphology of *Xenotheca* there is little or no

evidence which is at all decisive. On the whole we are inclined to regard it provisionally as a cupule, though we have not obtained any example of this organ either containing a seed or associated with seeds. On the other hand it is possible, of course, that it was a cup-shaped indusium enclosing sporangia, but if this was so it differs greatly in size and in other respects from any Palæozoic indusium which has so far been described. So far as it can be compared with Carboniferous cupules there is undoubtedly a certain resemblance. This is perhaps greatest in the direction of the fructification of *Alcicornopteris Zeilleri* Vaffier*, from the Lower Carboniferous rocks of France. The sterile frond of this species is recorded as having some resemblance to *A. convoluta* Kidst., a British Lower Carboniferous species in which the fertile fronds and organs are unknown.

Vaffier's specimens are, unfortunately, not very well preserved. They show however a dichotomously branched axis terminating in organs which he calls indusia, but which nowadays would no doubt be interpreted as cupules. Vaffier expressly compares these organs with *Calymmatotheca Stangeri* Stur†, now regarded as cupules. Here however the habit is different to the Devonian specimens, and the cupules are more deeply lobed. In neither the French nor the Austrian species do any seeds occur. Vaffier's type, with six teeth, is no doubt specifically distinct from the specimens described here, although to us the resemblance is otherwise fairly close.

There is also some similarity in habit to the British Coal Measure *Lagenostoma Sinclairi* Arb.‡ in which however the cupule is less deeply divided, and to the German Devonian type figured but not described by Unger§ in 1856, which appear to show several cupules bearing seeds. If this is the right interpretation of the latter specimen it is the oldest cupulate Pteridosperm known.

While we incline to the cupule-interpretation of *Xenotheca,* it should be pointed out that somewhat similar structures have been described as spore-bearing. One of these is *Codonotheca* from the Coal Measures of the United States, described by Sellards‖ and regarded by him as the male organ of a *Neuropteris.*

There is also a possible, but as it seems to us a much more remote comparison with the fructification attributed to

* Vaffier, *Étude Géol. et Pal. Carbon. Infér. Maconnais*, p. 124, pl. vi. fig. 5; pl. vii. figs. 1, 1*a*—1*b*, 1901.

† Stur, *Culm Flora*, Part 2, 1877, p. 151, text-fig. 27 on p. 158, pl. viii. figs. 5—7, 1877. See also more recent figures in Oliver, *Biol. Centralb.* Vol. xxv. p. 412, fig. 6, 1905.

‡ Arber, *Proc. Roy. Soc.* Vol. B 76, 1905, p. 251, pl. ii. fig. 11.

§ Unger *Denksch. K. Akad. Wissen. Wien* (Math.-Nat. Cl.), Vol. xi. 1856, p. 184 (explanation to plates), pl. vi. fig. 25.

‖ Sellards, *Amer. Journ. Sci.* Vol. xvi. 1903, p. 87; and *New Phytol.* Vol. vi. 1907, p. 175.

*Psilophyton** and more especially with certain figures given by Feistmantel† of specimens from Rothwaltersdorf, attributed to this genus.

Xenotheca, gen. nov. *Diagnosis.* Dichotomously branched somewhat slender axes, the finer branches each terminated by a large cupule-like organ, cup-shaped, and ending in several teeth.

Xenotheca devonica, sp. nov. *Diagnosis.* Axis dichotomising at least three times, ·5—1 mm. broad, the branches between the dichotomies or the cupules varying from 5 mm. to 3·5 cm. in length. Cupules, cuplike in form, 8 to 15 mm. or more in length, and 4 to 10 mm. in breadth, with, distally, usually eight lanceolate teeth, 2 to 4 mm. in length.

Telangium sp.

Plate IV, figs. 8, 9.

Two examples of isolated fructifications occur, which consist of tufts of sporangia united at the base and mounted on a common stalk. One of the specimens is figured, somewhat enlarged, on Plate IV, fig. 9 and shows several groups of sporangia. The other, fig. 8, shows a single tuft of six or more members, natural size. The sporangia are about 2 mm. in length, and in most cases there appear to be more than six in the tuft. They are free for the greater part of their length, but it is impossible to say whether they are united together at the base as a synangium, or are each inserted independently on the common stalk.

These specimens, although specifically distinct, have a close resemblance to the Lower Carboniferous *Sphenopteris* (*Telangium*) *affinis* L. and H.‡ and *S.* (*T.*) *bifida* L. and H.§

Knorria sp.

Text-fig. 3.

Several good examples of *Knorria*-casts were obtained by Townsend Hall from Sloly quarry, one of which is shown in text-fig. 3. These specimens are in the Atheneum Museum, Barnstaple. There are also others in Lt.-Col. Harding's collection at Jermyn Street, and in the Valpy Collection in the British Museum (Nat. Hist.). The last mentioned are from the Marwood

* Dawson, *Foss. Plants Dev. and Upper Silur. Canada*, Part 1, p. 39, pl. x. fig. 121, pl. xii. especially fig. 140, 1871.

† Feistmantel, *Zeitschr. Deut. Geol. Gesell.* Vol. xxv. 1873, p. 541, pl. xvii. figs. 39, 40.

‡ Peach, *Quart. Journ. Geol. Soc.* Vol. 34, pl. viii. figs. 1—3, 1878.

§ Kidston, *Trans. Roy. Soc. Edinb.* Vol. 33. pt. 1, 1887, pl. viii. figs. 1—6*a*.

Fig. 3. *Knorria* sp. from Sloly Quarry, N. Devon. From a specimen in the Atheneum Museum, Barnstaple (Hall Coll.), very slightly reduced.

beds at Plaistow, near Sherwell. A few examples are also in the Sedgwick Museum, Cambridge.

The figured specimen measures 21 cm. in length and 6·5 cm. in breadth. It is now known that *Knorria* is the cast of the inner surface of the outer cortex of a Palæozoic Lycopodiaceous stem. It is not however possible to identify the genus (e.g. *Bothrodendron*, *Lepidodendron*, etc.) from such specimens, and there is perhaps little to be gained by distinguishing species of *Knorria*. We are not aware that precisely similar specimens have been already described from Devonian rocks, but these fossils are very common in Lower Carboniferous sediments in various parts of the world. Among these, *Knorria imbricata*, Sternb., and still more *K. longifolia*, Schimp.*, the latter from the Vosges, may be compared with the Devonshire specimens.

It may be also mentioned here that although there are fragments of ridged casts in various collections, which recall portions of the internodes of *Asterocalamites*, we have not been able to satisfy ourselves that any real evidence of the occurrence of this genus in Devonshire has as yet been found. It is a well-known fact that imperfect casts of Lycopods in the *Knorria* condition may be easily mistaken for pith casts of *Asterocalamites*. Many of the specimens described by Heer from the Devonian of Bear Island, and attributed by him to the latter genus, are in reality, as Nathorst has shown, to be referred to *Bothrodendron*.

? *Cordaites* sp.

Plate V, fig. 5.

In the Townsend Hall collection in the Atheneum, Barnstaple, there is a fragment of a small leaf-like impression with parallel longitudinal striæ. This is figured on Plate V, fig. 5, and to some degree it recalls the more slender Cordaitean leaves of the Carboniferous rocks. The specimen was derived from the Marwood beds at Plaistow. It measures however only 2·2 cm. in length and 7 mm. across, and is thus too fragmentary to be regarded as satisfactory evidence of the occurrence of *Cordaites* in the Devonian of North Devon.

This is the most satisfactory specimen which we have seen from this locality. Among other fragments, more obscure and indistinct, there is one from Plaistow (Plate V, fig. 7) in the Valpy collection (V. 10682) British Museum (Nat. Hist.) which has some resemblance to a cone and to a *Lepidostrobus*, but which is in reality too imperfect to merit description or determination.

* Schimper, *Terr. Trans. Vosges*, 1862, pl. xiv. upper fig. etc.

Obscure Axes.

In addition to the dichotomously branched axes of *Xenotheca*, several specimens of naked axes, laterally branched, have been obtained at Baggy Point. There is also at least one example of a cymosely branched type, somewhat recalling *Hostimella* from the Devonian of Bohemia, but this is too fragmentary to allow of any real comparison with that genus. The affinities of these axes remain wholly obscure.

Conclusions.

With the exception of the obscure plant remain described from the Lynton beds, all the other determinations here recorded relate to terrestrial plants from the Baggy or *Cucullæa* beds of the Upper Devonian of North Devon. These are:

Sphenopteridium rigidum (Ludw).
Sphenopteris sp.
Xenotheca devonica gen. et spec. nov.
Telangium sp.
Knorria sp.
Cordaites? sp.

We can find no evidence of the occurrence of *Archæopteris hibernica* (Forbes) in Devonshire; and so far as we are aware the only valid determinations among previous records are included in the above list.

Though the number of records is small, these specimens are of particular interest as being the oldest (in a geological sense) terrestrial plants known from England. The occurrence of a cupulate organ, *Xenotheca*, which is probably the first to be demonstrated in rocks of Devonian age, is of importance as tending to confirm the conclusion that the Pteridosperms were an important group even at this early period.

On the vexed question as to whether the higher part of the so-called Devonian sequence in North Devon, to which these specimens belong, should not be referred to the Lower Carboniferous, the known flora of the beds in question sheds hardly any light. There are in the first place good grounds for the belief that the Lower Carboniferous flora is very closely related, and in many respects similar to that of the Upper Devonian, and supposing that the Baggy beds really are of Devonian age, as they may well be, the specimens described here tend to confirm this conclusion. We should at any rate not expect to find in Devonshire, in the higher beds of the Devonian, a flora markedly dissimilar from that

of the British Lower Carboniferous rocks elsewhere, nor indeed is this the case.

Of the two species described here, one is a new type and the other is a plant only known from the Devonian. The other genera recorded and the particular types themselves are similar to those occurring in the Lower Carboniferous. On the other hand there is no plant represented which is particularly characteristic of the Lower Carboniferous. We therefore conclude that on the whole this flora is probably of Devonian age, or at least that there is no evidence to be gained from the specimens described here which is contrary to this view.

EXPLANATION OF PLATES.

Plate IV.

Fig. 1. *Xenotheca devonica* gen. et spec. nov. Two thecae terminating a dichotomous axis. These thecæ bear large scars. From Croyde Hoe, Baggy Point, N. Devon. Baggy Beds. No. 103, Brit. Devonian Plant Coll. Sedg. Mus. Camb. ×3.

Fig. 2. *Xenotheca devonica*, the same specimen as fig. 1. Nat. size.

Fig. 3. *Xenotheca devonica*? Probably a badly preserved theca. From the same locality and No. 91 in the same collection. ×$\frac{3}{2}$.

Fig. 4. Scars on the thecæ seen in figs. 1 and 3, enlarged. ×8.

Fig. 5. *Xenotheca devonica*. A theca. From the same locality and No. 92 in the same collection. Nat. size.

Fig. 6. *Xenotheca devonica*. A well preserved theca. From the same locality and No. 96 in the same collection. ×$\frac{3}{2}$.

Fig. 7. *Xenotheca devonica*. A theca in which the teeth are well preserved. From the same locality and No. 106 in the same collection. ×$\frac{5}{2}$.

Fig. 8. *Telangium* sp. A tuft of sporangia. From the same locality and No. 129 in the same collection. Nat. size.

Fig. 9. *Telangium* sp. Tufts of sporangia. From the same locality and No. 122 in the same collection. ×$\frac{3}{2}$.

Fig. 10. *Xenotheca devonica*. A habit specimen showing the dichotomous branches ending in badly preserved thecæ. From the same locality and No. 106 in the same collection. Nat. size.

Fig. 11. *Xenotheca devonica*, the same specimen as fig. 7. Nat. size.

Fig. 12. *Sphenopteridium rigidum* (Ludw.). Fragment of a pinnule showing traces of the nervation. From the same locality and No. 116 in the same collection. ×$\frac{5}{2}$.

Plate V.

Fig. 1. *Sphenopteridium rigidum* (Ludw.). From Sloly Quarry, Barnstaple. V. 3562, British Museum (Nat. Hist.), Hall Coll. Very slightly reduced.

Fig. 2. *Sphenopteris* sp. from Croyde Hoe, Baggy Point, North Devon. Baggy Beds. No. 109, Brit. Devonian Plant Coll. Sedg. Mus. Camb. ×3.

Fig. 3. *Sphenopteridium rigidum* (Ludw.). Lobes of a pinnule enlarged. From the same locality and No. 113 in the same collection as fig. 2. ×3.

Fig. 4. *Sphenopteris* sp. From the same locality and No. 110 in the same collection as fig. 2. $\times\frac{10}{3}$.

Fig. 5. *Cordaites*? sp. from the Marwood beds at Plaistow, North Devon. Hall Coll. Atheneum Museum, Barnstaple. ×2.

Fig. 6. *Sphenopteridium rigidum* (Ludw.). Enlarged pinnules. From the same locality and No. 117 in the same collection as fig. 2. $\times\frac{10}{3}$.

Fig. 7. Obscure cone from the Marwood beds at Plaistow, North Devon. V. 10682, British Museum (Nat. Hist.), Valpy Coll. Nat. size.

3 x$\frac{3}{2}$

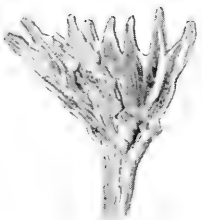
2

4 x8

1 x3

5

8



7 x$\frac{5}{2}$

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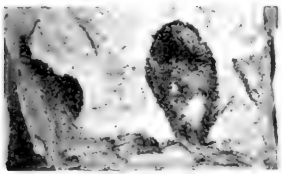
T. A. Brock del.

Upper Devonian Fossil Plants.



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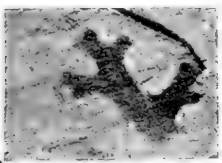

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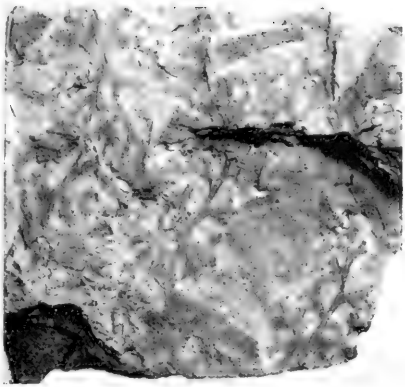

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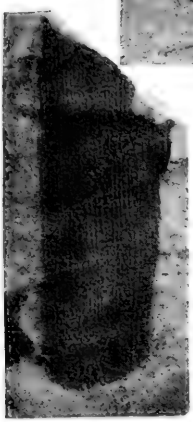

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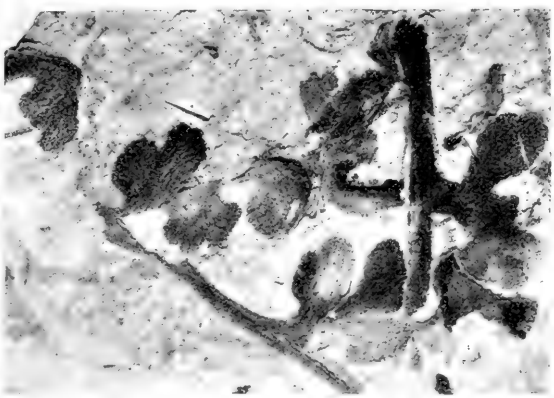

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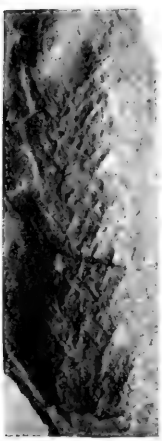

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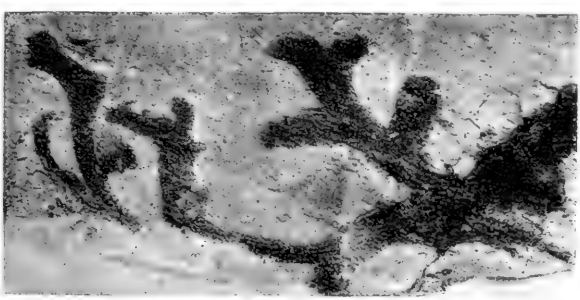


6 x3

W. Toms Photo.

Upper Devonian Fossil Plants.

On some new and rare Jurassic plants from Yorkshire: The male flower of Williamsonia gigas (*Lind. and Hutt.*). By H. Hamshaw Thomas, M.A., Fellow of Downing College, Curator of the Botanical Museum, Cambridge.

[Plate VI.]

[*Read* 22 February 1915.]

Recent years have seen considerable additions to our knowledge of the reproduction structures (often called 'flowers') of the *Williamsonia* section of the Bennettitales, but several points still require elucidation, even in the case of *Williamsonia gigas* which was the first to be discovered and described. The classic work of Williamson* published in 1870 gave an account of the female strobilus, and the beautiful specimens on which the paper was based are now in the possession of Prof. Seward and have been therefore specially accessible for study. More recently Prof. Lignier, working from the specimens in the Yates collection in the Museum of Natural History at Paris, reinvestigated this species† and added much to our knowledge and to the correlation with other Bennettitalean flowers. The male sporophylls, however, remained unknown, Lignier acknowledging that we had no information as to their form or position‡. In 1909, Prof. Nathorst§ described some male Williamsonian flowers from Whitby, which were found as isolated carbonaceous impressions and which he called *Williamsonia spectabilis*; subsequently he also described several other species from the same district‖. All these forms were cup-shaped structures composed of partially united microsporophylls, bearing synangia from which the remains of pollen-grains could be extracted; they were probably unisexual 'flowers.' Other observations have since been made, and I have collected a number of specimens of various forms¶, but the question as to the male structures in *W. gigas* still remains to be answered.

Wieland has maintained that one of the specimens figured by Lignier as the cast of the apical part of an ovulate strobilus is

* "Contributions towards the History of *Zamia gigas*, Lind. and Hutt." *Trans. Linn. Soc.* Vol. xxvi. p. 663, 1870.

† "Le fruit du *Williamsonia gigas* Carr. etc." *Mém. Soc. Linn. de Normandie*, T. xxi. p. 19, 1903.

‡ *idem*, p. 43.

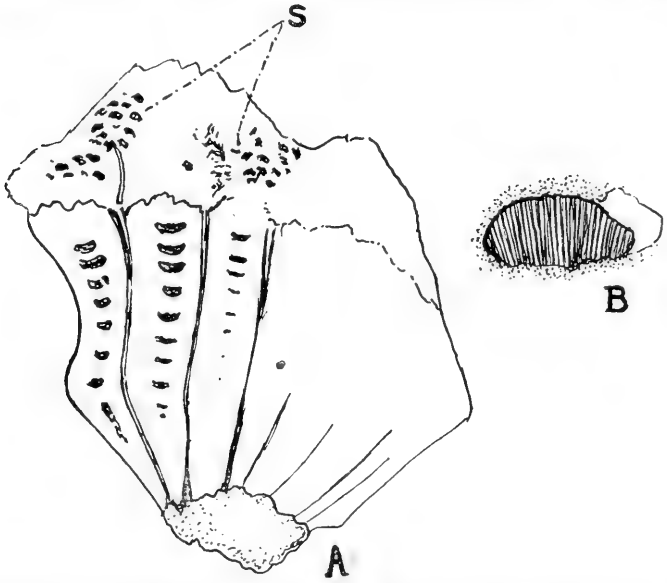
§ "Paläobotanische," Mitteilungen 8. *K. Vet. Akad. Hand.* Bd. 45. Stockholm, 1909.

‖ "Paläobotanische," Mitteilungen 9. *K. Vet. Akad. Hand.* Bd. 46. Stockholm, 1911.

¶ "Fossil flora of Cleveland district." *Q. J. G. S.* Vol. lxix. p. 230, 1913.

really the apical part of a bud in which the ovulate structures were surrounded by closely packed microsporophylls on which the remains of microsporangia could be seen*. Examination of this specimen and comparison with other examples of the same type, leave no doubt as to the correctness of Lignier's interpretation.

But while discussion and speculation were proceeding about these forms, a specimen in the Yates collection in the Paris museum remained unnoticed, although it presented the cast of an indubitable male flower, and probably belonged to the species



Text-fig. 1. A. Diagrammatic drawing of specimen showing position of the synangia at *S* and higher parts of the sporophylls. B. Drawing of a single synangium. ×6.

under discussion. While working in Paris in July, 1914, I was enabled by the kindness of Dr Pelourde to examine carefully the specimens in the Yates collection and there found this flower, which is figured on Pl. VI and described below.

The specimen was unlabelled, but consisted of a piece of red ironstone in which the plant appeared as a mould or cast; some of the original tissues are left as faint carbonaceous markings which are associated with patches of white Scarbroite, and the example undoubtedly originated from the cliffs of Runswick or Hawsker

* *American Fossil Cycads*, Washington, 1906, p. 152, and "The Williamsonian Tribe," *Amer. Journ. Sci.* Vol. xxxii. p. 463, 1911.

in the neighbourhood of Whitby, from which all the other examples of *W. gigas* have been obtained.

One side of the specimen shows a basin or urn-shaped form, while above this, in the irregular part of the matrix, the remains of synangia are seen (cf. text-fig. 1 A).

The 'flower' seems then to have been composed of 18–20 microsporophylls, each 7–8 mm. wide, united to form a cup-like structure 5–6 cms. wide. The base of the cup has been irregularly broken away, but tapered considerably, perhaps to a stalk no more than 1 cm. wide. Down the centre of each sporophyll forming the cup, was a series of closely approximated conspicuous depressions, elliptical or reniform in shape; they sometimes appear to lie in a groove and give the flower a very characteristic appearance. As we cannot be quite sure whether the example now seen is a mould of the inner surface of the original flower, it is somewhat difficult to arrive at an interpretation of these structures, but they are probably to be compared with the depressions figured by Prof. Nathorst in *W. whitbiensis* as corresponding to rudimentary synangia; in the present case however, only a single row of these depressions is seen.

The shape of the cup formed by the united sporophylls calls for some notice. It is considerably different from the forms already described and may be made out from Pl. VI, fig. 1 and from the text-fig. 2. The sporophylls spread outwards from the base, but then curve round inwards as they continue, curving outwards again as they become free; in this way a very characteristic ridge is formed which runs round the periphery of the cup.

The upper parts of the sporophylls are not so clearly seen. In most cases the free portions have been broken away, but in the view shown in text-fig. 2, part of a sporophyll seems to continue upwards for about 2–3 cms., and there are indications that branches were given off in this region which projected inwards towards the centre of the flower. If this were really the case, the flower must have shown considerable similarity to that of *W. spectabilis* Nath.* It is impossible to determine how the synangia were produced, though the remains of many of these structures are seen in the matrix. Their irregular position in the rock supports the conclusion that they were formed in the manner seen in *W. spectabilis* rather than as in *W. whitbiensis*. The best preserved synangia show the usual reniform shape, text-fig. 1 B, and a large number of fine striations traversing them. All traces of their original substance have disappeared,

* Cf. my restoration, text-fig. 2, *Q.J.G.S.* 1913, p. 231, and Nathorst's in "Die Microsporophylle von Williamsonia" *Arkiv. för Botanik*, Bd. 12, No. 6, p. 7, 1912.

but the fine grain of the ironstone matrix has retained their external form very clearly.

In assigning this flower to its systematic position we may first notice that it agrees closely with the Williamsonian male flowers previously known, and there can be little or no doubt about its generic position. As to its specific name, there is no direct evidence, but we have the following considerations:

(*a*) The specimen comes from a bed in which stems, leaves and female flowers of *Williamsonia gigas* occur frequently, and from which other species of *Williamsonia* are practically absent. (*b*) It is to be expected that the male flowers would occur in close proximity to female flowers, stems and leaves of the same species. (*c*) The flower here described differs specifically from other species of the genus which are found in the same neighbourhood; four

Text-fig. 2. Diagrammatic drawing of the specimen from the side seen in Pl. VI, fig. 1. The lower part of the united sporophylls and indications of the upper portion of a single one are shown.

other species of male flowers and three or four species of female strobili have now been found in the Lower and Middle Estuarine Shales. There is therefore a very strong presumption that the present flower belonged to the plants of *Williamsonia gigas* (L. and H.), unless strong evidence to the contrary should be forthcoming.

Little can be said as to the position and mode of growth of this structure. Its homologies with *W. spectabilis* are clear, and it, no doubt, had the same position and origin as seen in that form. There are three alternative views as to the original position of these male flowers. One view would regard them as originating in a bisexual flower of the same type as that of *Cycadeoidea*

ingens with the male sporophylls borne below the female strobilus. This may be regarded as the normal type of bisexual structure in the Bennettitales, but I do not consider the present specimen as conforming to it.

In some specimens of the female flowers of *W. gigas* we see the female strobili still attached to the peduncle and surrounded by many overlapping bracts, without any trace of our cup-shaped microsporophylls. Had such a structure been originally present it must have become loosened from the axis and slipped over the ovuliferous portion, becoming free and independent without breaking up. But there are no signs that the bases of these cup-like united microsporophylls separated or broke up to allow them to slip over the female strobilus. If the analogy with *W. spectabilis* is at all complete, our male flower would have possessed a somewhat narrow stalk and the type of origin just suggested would be impossible.

Moreover the curious urn-shape of the cup suggests a structure which was freely exposed during its development and not enclosed by ensheathing bracts in the circular buds so well known in this species. There are in the matrix of our specimen no traces of bract-like structures, and had any number of them been originally present, the longitudinal section through the male flower would have probably presented a more or less circular form.

The second view of the possible origin of the microsporophylls would see them as arising above the ovulate portion of the flower and in the position in which Lignier has placed his 'appendice infundibuliforme.' There are however serious objections to this view. In the first place the bisexual Bennettitalean flowers already known, i.e. *Cycadeoidea*, *Wielandiella* and *Williamsoniella*, all have their microsporophylls below the megasporophylls, and no specimens are known in which the microsporophylls were produced in the superior position. In the second place the evidence for the existence of such a structure as Lignier's infundibuliforme appendage is very meagre, and I have myself seen no specimen which shows any trace of such a structure. The apex of the strobilar axis without such an appendage may be closely compared with that of the new *Williamsoniella* and also with an undescribed species of *Williamsonia* in my collection. Another objection which may be tentatively advanced, is that a flower of *Williamsonia*, with the male disc attached above the seed zone, would be a very top-heavy and rather uneconomical form.

The third view of the origin of the structure here described is that it was a separate 'flower' produced on its own stalk, and independent of the female strobilus. The unisexual theory of the flowers fits in much better with the known facts than the other

theories and involves fewer assumptions than do the contrary views. These unisexual flowers may have originated from bisexual flowers, and the form *Wielandiella* which has been described by Prof. Nathorst* doubtless indicates a stage in reduction of a completely bisexual flower to one in which the ovulate portion only was fully developed, the microsporophylls being very much reduced and delayed in development. I therefore regard the structure which has been described above as an independent male flower of *Williamsonia gigas* (L. and H.).

In conclusion I must express my best thanks to Dr Pelourde of the Paris Museum for his kindness in enabling me to study and photograph the specimen, and to Prof. Seward to whose encouragement the appearance of this note is due.

DESCRIPTION OF FIGURES.

Plate VI.

Fig. 1. Specimen of male flower of *W. gigas* in the Yates collection, Natural History Museum, Paris. Photographed from one side showing basal part of united sporophylls, and on the right side above, the region in which the synangia were borne, cf. text-fig. 2.

Fig. 2. Photograph of the same specimen from below, showing circular outline of cup. The base is not distinctly seen, but the sporophylls each with a central row of large depressions are seen on the left side.

* "Paläobotanische," Mitteilungen 8. *ibid.*

Fig. 1.

H. H. T. phot.

Fig. 2.

Williamsonia gigas (Lind. and Hutt.).

Calculation of the electrical resistance of a certain network of conductors. By G. F. C. SEARLE, Sc.D., F.R.S., University Lecturer in Experimental Physics, Fellow of Peterhouse.

[*Read* 8 February 1915.]

§ 1. The system of conductors is thus described in a question set in the Mathematical Tripos, Part I, 1914:

"Two uniform cables $A_0A_1 \ldots$, $B_0B_1 \ldots$ are divided into parts A_0A_1, $A_1A_2, \ldots$, B_0B_1, $B_1B_2, \ldots$, each of resistance r, and wires, each of resistance s, are used to join A_1 to B_1, A_2 to B_2, ... When the cables are joined by the n wires A_1B_1, A_2B_2, ... A_nB_n, the resistance of the system from A_0 to B_0 is R_n."

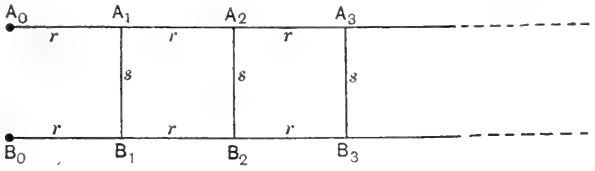


Fig. 1.

Fig. 1 shows the system when $n = 3$.

In order to obtain the value of R_n as a function of n it is necessary to obtain a formula of reduction by which R_n is found in terms of R_{n-1}. The necessary electrical equation is very easily found, if it is noted that the resistance between A_1 and B_1 when the cables are connected only by the $n-1$ wires A_2B_2, $A_3B_3, \ldots A_nB_n$ is equal to R_{n-1}. Then R_n is the resistance between A_0 and B_0 in the system shown in Fig. 2, where A_1 and B_1 are joined by the

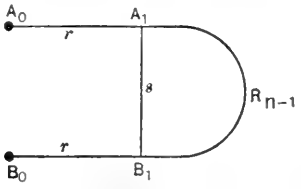


Fig. 2.

two conductors s and R_{n-1} "in parallel." Hence we find, what candidates were asked to prove, that

$$R_n = 2r + \frac{sR_{n-1}}{s + R_{n-1}}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

The method employed in obtaining the value of R_n as a function of n is indicated by Bromwich (*Infinite Series*, p. 21, Example 26). We have, from (1),

$$R_n - x = \frac{R_{n-1}(2r+s-x)+s(2r-x)}{s+R_{n-1}}$$

$$= \frac{(R_{n-1}-x)(2r+s-x)-(x^2-2rx-2rs)}{s+R_{n-1}}. \quad \ldots\ldots(2)$$

The second term in the numerator vanishes if

$$x^2 - 2rx - 2rs = 0, \quad \ldots\ldots(3)$$

or if $$x = r \pm \sqrt{r^2+2rs} = r \pm t. \quad \ldots\ldots(4)$$

Since there are *two* values of x satisfying the condition (3), we obtain the *two* equations

$$R_n - r - t = \frac{(R_{n-1}-r-t)(r+s-t)}{s+R_{n-1}}, \quad \ldots\ldots(5)$$

$$R_n - r + t = \frac{(R_{n-1}-r+t)(r+s+t)}{s+R_{n-1}}. \quad \ldots\ldots(6)$$

Putting $$p = r+s+t, \quad q = r+s-t, \quad \ldots\ldots(7)$$

so that $$r+t = p-s, \quad r-t = q-s, \quad \ldots\ldots(8)$$

we obtain from (5) and (6)

$$\frac{R_n+s-p}{R_n+s-q} = \frac{R_{n-1}+s-p}{R_{n-1}+s-q}\left(\frac{q}{p}\right). \quad \ldots\ldots(9)$$

This is the required formula of reduction. Since $R_1 = 2r+s$, we obtain by successive applications of (9)

$$\frac{R_n+s-p}{R_n+s-q} = \frac{2r+2s-p}{2r+2s-q}\left(\frac{q}{p}\right)^{n-1} = \left(\frac{q}{p}\right)^n. \quad \ldots\ldots(10)$$

Solving (10) for R_n, we have

$$R_n = \frac{p^{n+1}-q^{n+1}}{p^n-q^n} - s, \quad \ldots\ldots(11)$$

which is the complete solution of the problem.

§ 2. We can write (11) in the form

$$R_n = \frac{p-q(q/p)^n}{1-(q/p)^n} - s. \quad \ldots\ldots(12)$$

Since $p > q$, it follows that $(q/p)^n$ vanishes when n becomes infinite. Hence, if R_∞ be the resistance between A_0 and B_0 when the number of connecting wires between the cables is infinite,

$$R_\infty = p - s = r + t = r + \sqrt{r^2+2rs}. \quad \ldots\ldots(13)$$

This last result could have been obtained at once from (1). For R_n is always greater than $2r$ though less than $2r+s$ when $n>1$, and R_n diminishes as n increases. Hence R_n has a limit R_∞ when n becomes infinite. Putting $R_n = R_\infty$ and $R_{n-1} = R_\infty$ in (1), we have

$$R_\infty = 2r + \frac{sR_\infty}{s+R_\infty}.$$

Since R_∞ is essentially positive, this quadratic gives

$$R_\infty = r + \sqrt{r^2+2rs},$$

which is identical with (13).

§ 3. Mr J. M. Dodds, Fellow of Peterhouse, has kindly drawn my attention to the advantages of the use of hyperbolic functions in this problem. Since

$$\sinh(n+1)\theta + \sinh(n-1)\theta = 2\cosh\theta \sinh n\theta$$

and

$$\sinh 2\theta = 2\sinh\theta\cosh\theta,$$

we have

$$\frac{\sinh(n+1)\theta}{\sinh n\theta} = \frac{\sinh 2\theta}{\sinh\theta} - \frac{\sinh(n-1)\theta}{\sinh n\theta}. \quad \text{.....(14)}$$

Now, since $R_1 = 2r+s$, (1) can be written

$$R_n + s = 2r + 2s - \frac{s^2}{R_{n-1}+s} = R_1 + s - \frac{s^2}{R_{n-1}+s},$$

or

$$\frac{R_n+s}{s} = \frac{R_1+s}{s} - \frac{s}{R_{n-1}+s}. \quad \text{.....(15)}$$

Comparing (15) with (14) we see that, if θ be suitably chosen,

$$R_n + s = s\frac{\sinh(n+1)\theta}{\sinh n\theta}. \quad \text{.....(16)}$$

When $n=1$, this gives, since $R_1 = 2r+s$,

$$\cosh\theta = (r+s)/s.$$

Since

$$\sinh\theta = \sqrt{\cosh^2\theta - 1} = \frac{1}{s}\sqrt{r^2+2rs} = \frac{t}{s},$$

we have

$$e^\theta = \cosh\theta + \sinh\theta = \frac{1}{s}(r+s+t) = \frac{p}{s},$$

and

$$e^{-\theta} = \cosh\theta - \sinh\theta = \frac{1}{s}(r+s-t) = \frac{q}{s}.$$

Hence

$$R_n + s = s\frac{\sinh(n+1)\theta}{\sinh n\theta} = s\frac{e^{(n+1)\theta} - e^{-(n-1)\theta}}{e^{n\theta} - e^{-n\theta}} = \frac{p^{n+1} - q^{n+1}}{p^n - q^n},$$

which agrees with (11).

§ 4. The currents in the cables and in the wires can now be easily found. Let the E.M.F. applied between A_0 and B_0 be E, let the current along A_0A_1 be x_1, and that along $A_{m-1}A_m$ be x_m. Let the current in the wire A_mB_m be y_m. Since the resistance of the system beyond A_mB_m is R_{n-m}, we have

$$sy_m = R_{n-m}x_{m+1}. \quad \text{.....(17)}$$

The "continuity" of the current gives

$$x_{m+1} = x_m - y_m. \quad \text{.....(18)}$$

From (17) and (18) we have, by (16),

$$x_m = \frac{R_{n-m}+s}{s}\, x_{m+1} = \frac{\sinh(n-m+1)\theta}{\sinh(n-m)\theta}\, x_{m+1}.$$

Hence

$$\frac{x_{m+1}}{\sinh(n-m)\theta} = \frac{x_m}{\sinh\{n-(m-1)\}\theta} = \dots = \frac{x_1}{\sinh n\theta},$$

so that

$$x_m = \frac{\sinh(n-m+1)\theta}{\sinh n\theta}\, x_1. \quad \text{.....(19)}$$

But $x_1 = E/R_n$, and hence, by (19) and (16),

$$x_m = \frac{\sinh(n-m+1)\theta}{\sinh(n+1)\theta - \sinh n\theta}\cdot\frac{E}{s}.$$

Since $2\sinh^2\frac{1}{2}\theta = \cosh\theta - 1 = r/s$, we have

$$x_m = \frac{\sinh(n-m+1)\theta}{\cosh(n+\frac{1}{2})\theta}\cdot\frac{E}{\sqrt{2rs}}.$$

Since, by (18), $y_m = x_m - x_{m+1}$, we have

$$\begin{aligned} y_m &= \frac{E}{\sqrt{2rs}\,.\cosh(n+\frac{1}{2})\theta}\{\sinh(n-m+1)\theta - \sinh(n-m)\theta\} \\ &= \frac{2E\cosh(n-m+\frac{1}{2})\theta\sinh\frac{1}{2}\theta}{\sqrt{2rs}\,.\cosh(n+\frac{1}{2})\theta} \\ &= \frac{\cosh(n-m+\frac{1}{2})\theta}{\cosh(n+\frac{1}{2})\theta}\cdot\frac{E}{s}. \end{aligned}$$

The determination of the focal length of a thick mirror. By G. F. C. SEARLE, Sc.D., F.R.S., University Lecturer in Experimental Physics, Fellow of Peterhouse.

[*Read* 8 February 1915.]

§ 1. *Introduction.* The experiments described in the following paper bear the same relation to those made upon an ideal spherical mirror as the measurement of the focal length of a thick lens or of a lens system bears to the measurement of the focal length of an ideal thin lens. They thus form a useful introduction to experimental work with lens systems.

The properties of mirror systems are simpler than those of lens systems, since a lens has two foci, two principal points and two nodal points, while a mirror has only one focus, one principal point and one nodal point, the latter being what is called the "centre" of the mirror system in § 4.

The first method (§ 5) was introduced into my practical class at the Cavendish Laboratory in 1908; the other two methods (§§ 7, 9) were devised after the paper was "read."

§ 2. *Ideal mirror.* When a concave spherical mirror is formed of metal or of polished silver deposited on the front surface of a spherical surface of glass, the reflexion takes place at the *surface*. If u and v be the distances of an object point P and its image Q from the vertex, i.e. the point in which the surface of the mirror is cut by the straight line through the two points,

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

where the constant f is called the focal length of the mirror. Here u and v are counted positive when P and Q are on the same side of the vertex as the centre of the spherical surface. If we make u infinite, then $v=f$, so that f is the distance from the vertex to the point to which parallel incident rays are reflected to a focus.

If we put $u=0$ in (1), we find $v=0$, and thus the vertex is a self-conjugate point.

If we put $u=2f$, we find that $v=2f$, and thus

$$u=v=2f$$

gives a second self-conjugate point. Hence, if we place an object point in such a position (not in contact with the mirror) that it coincides with its own image, the distance of the point from the mirror is $2f$.

It is clear that the point is in this case at the centre of the spherical surface, and hence, if r be the radius of the sphere,

$$r = u = v = 2f.$$

There is an important distinction between the two self-conjugate points, for at the point where $u = v = 0$, the image of a finite object is *erect*, but at the point where $u = v = 2f$, the image is *inverted*.

§ 3. *"Thick" mirrors.* Many mirrors are formed of glass silvered at the back, the glass having two spherical surfaces which are generally not concentric. Although the front (unsilvered) surface acts as a spherical mirror, the images formed by it are very weak compared with those which are formed by two refractions at the front surface and one reflexion at the silvered surface. The more general system is described in § 4.

We see at once that we must not expect a thick mirror to act exactly as if the front surface of the glass were silvered and polished. Yet we can show that the positions of image and object are related in the same way as if the glass mirror were replaced by an ideal spherical mirror of appropriate radius placed in the proper position relative to the front surface of the actual system, under the limitation that any object point is very close to the axis of the system and that the rays make only infinitesimal angles with that axis.

§ 4. *Theory of the thick mirror.* The mirror may consist of a piece of glass bounded by two spherical surfaces and silvered at the back, or it may be a more complicated arrangement consisting of any number of thick or thin lenses arranged along an axis, the reflexion taking place at a plane or spherical silvered surface situated behind the last of the lenses.

In Fig. 1, AC is the axis of the spherical surfaces. Let $P'X$ be an incident ray parallel to the axis. When this ray emerges

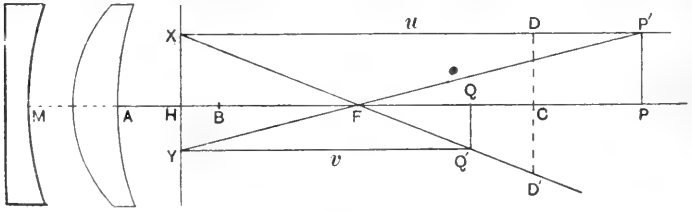


Fig. 1.

from the system after suffering reflexion at the silvered surface M, it will not, in general, be parallel to the axis. It will, therefore, intersect the incident ray in some point X and will cut the axis

in some point F. Draw a plane XH at right angles to the axis, cutting the axis in H. Then, if we reverse the ray FX, we have the two incident rays $P'X$ and FX giving rise to the emergent rays XF and XP', and thus X is its own image. From the general properties of refractions and reflexions at spherical surfaces, it follows that *every* point near H in the plane XH coincides with its own image or, in other words, is self-conjugate.

The point conjugate to F is at infinity, since the ray XP' is parallel to the axis, and hence *every* ray which passes through F on incidence and makes a small angle with the axis is parallel to the axis on emergence.

The point F is called the Focus of the system and H is called the Principal or Unit Point. The plane HX is called the Principal or Unit Plane of the system.

We can now find the image of the point P'. Let the incident ray $P'F$ cut the principal plane in Y. Since Y coincides with its own image, the emergent ray passes through Y. This ray YQ' is parallel to the axis, since the incident ray $P'FY$ passed through the focus F. The image of P' is therefore at Q' where XF and YQ' intersect.

If P and Q be the feet of the perpendiculars from P' and Q' upon the axis, then P and Q are conjugate points.

Let u and v be the distances of P and Q from the principal plane; let $HF=f$ and let u, v and f be counted positive when in each case a point moving in the direction of the normal drawn *from* M reaches H before it reaches P, Q and F respectively. Then

$$\frac{f}{u}=\frac{YH}{YX} \text{ and } \frac{f}{v}=\frac{XH}{XY}.$$

Hence, by addition,
$$\frac{f}{u}+\frac{f}{v}=1,$$

or
$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Comparing (2) with (1), we see that the mirror system may be replaced by an ideal spherical mirror of focal length f, if the reflecting surface of the latter intersects the axis at right angles in H. We shall show presently how f and the position of H may be found by experiment.

If we put $u=2f$, we find from (2) that

$$u=v=2f,$$

and thus the object point P and its image Q coincide. If C (Fig. 1) be the corresponding point, then the focus F is midway between C and H.

There are thus two points on the axis which are self-conjugate, viz. H and C, and if we were to use merely a *point* as an object we could not determine by experiment which of the two points was H and which was C. But the image of a finite object in the transverse plane through C lies in that plane and is inverted. This follows from the fact that, if the transverse plane through C cut $P'X$ in D and XF in D', D and D' are on opposite sides of C. Since $HF = FC$ and since XD is parallel to the axis, $CD' = CD$.

The point C corresponds to the centre of the ideal spherical mirror and may be spoken of as the centre of the optical system.

On the other hand the image of an object in the principal plane HX is erect*.

Since $CF = f$, the focal length could be found by measuring CF. The point C is easily found by adjusting a pin so that its tip coincides with the tip of its own *inverted* image, and the point F could be found by means of a small screen of ground glass, for distant objects will be focused at F; illumination troubles would, however, make this plan difficult†.

Since the position of the centre C can be readily determined, we shall transform (2) so that we can use C as an origin of measurement. Let p and q be the distances of P and Q from C, and let p and q be positive when P and Q lie on the same side of C as F. Then

$$u = 2f - p, \quad v = 2f - q.$$

Hence, by (2),

$$\frac{1}{2f-p} + \frac{1}{2f-q} = \frac{1}{f}.$$

This equation leads to

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}. \qquad \text{.....(3)}$$

Thus, if we find two conjugate points P and Q and measure their distances from the centre C, we can find f. The positions of F and H follow at once, since $CF = f$ and $CH = 2f$.

§ 5. *Gauss's method.* The following method of finding f may be called Gauss's method, as it corresponds for mirrors to Gauss's method of finding the focal length of lenses. The centre C (Fig. 1) is found by adjusting a pin so that its tip coincides with the tip

* Mr R. E. Baynes has pointed out to me that, in the case of any general mirror system, the vertex M (Fig. 1) of the reflecting surface and the centre of curvature of that surface are the images of H and C respectively formed by the lens system in front of M in the medium in contact with M.

† If a telescope with cross-wires or a goniometer (§ 9) is available, the position of the focal plane is easily found. The instrument is first focused for infinity and is then turned to view the image of an object seen by reflexion in the mirror system. A grain of lycopodium on that face of a glass plate which is turned towards the mirror would serve excellently. The object is adjusted so that an image of it is focused (without parallax) upon the cross-wire of the telescope or goniometer.

of its own inverted image; the coincidence is tested by the parallax method, a magnifying lens being used. For accurate work the pin should be sharp and should be firmly supported. The distance of the pin's tip from the vertex A is then measured. An adjustable distance piece may be used or a short rod of known length may be attached to a steel scale so that its axis coincides with one edge of the scale. The ends of the rod should be rounded. We could, of course, use *any* point P and its conjugate Q to give us a value of f through (3), but it is convenient to take the vertex A (Fig. 1), in which the axis cuts the first surface, as one conjugate point. Some grains of lycopodium are placed on the surface; let one grain be at A. The system will form an image of this grain at some point B. The distance AB is measured

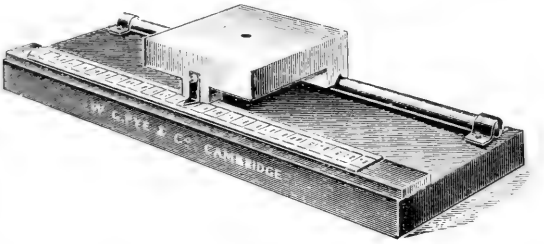


Fig. 2.

by a microscope attached to a sliding carriage (Fig. 2), the axis of the microscope being parallel to the direction of motion of the carriage. A vertical steel rod is fitted to the carriage and the microscope is attached to the rod by a suitable clamp; Pye's "Ideal" clamp is convenient. The microscope is first focused on A and the carriage is then moved until B, the image of A, is seen sharply in focus through the microscope. The distance AB is equal to the distance through which the carriage has been moved.

The microscope should be fitted with cross-wires or with a micrometer scale and the absence of parallax between the image of the lycopodium and the wire or scale is used as a test of correct adjustment. The distance between the object glass and the eyepiece of the microscope when B is observed must, of course, be the same as when A is observed.

Care must be taken to distinguish between A and its image. This may be done by holding an object such as a pointed piece of paper against the surface and then viewing the image with both eyes open. The binocular vision will at once decide whether B is in front of A or behind it. The one-eyed may hold a piece of paper in contact with the surface and may then focus the microscope on the surface of the paper. If the paper is large enough to

prevent any view, through the microscope, of its image, the scale reading of the microscope carriage corresponds to A and not to B.

The observations are facilitated by proper illumination of the grains. The best illumination is obtained if a strong beam of light is made to fall upon the grains in a direction nearly tangential to the surface of the mirror.

Let $AB = x$ and let x be positive when B lies in front of A. Then, if $AC = p$, we have $BC = p - x$, and thus, by (3),

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{p-x}. \quad \text{.....(4)}$$

Hence

$$f = \frac{p\,(p-x)}{2p-x} \quad \text{.....(5)}$$

Thus f is nearly equal to $\frac{1}{2}p$ when x/p is small.

If the distance of H from A be h, and if h be counted positive when H lies *in front of* A, we have

$$h = p - 2f = \frac{px}{2p-x}. \quad \text{.....(6)}$$

Thus h is nearly equal to $\frac{1}{2}x$ when x/p is small.

§ 6. *Practical example.* The following results were obtained in an experiment by Mr Heath upon an ordinary glass concave mirror silvered at the back.

Distance of centre C from vertex A, 19·70, 19·65, 19·65 cm.

Hence $p = 19{\cdot}67$ cm.

Reading of carriage when microscope is focused on lycopodium at A, 1·27, 1·26 cm. Mean reading 1·265 cm.

Reading of carriage when microscope is focused on image of A, 0·32, 0·31, 0·30 cm. Mean reading 0·310 cm.

The image of A was *behind* A. Hence

$$x = -(1{\cdot}265 - 0{\cdot}310) = -0{\cdot}955 \text{ cm.}$$

Then, by (5),

$$f = \frac{p\,(p-x)}{2p-x} = \frac{19{\cdot}67 \times 20{\cdot}625}{40{\cdot}295} = 10{\cdot}07 \text{ cm.}$$

To find the position of the principal point H relative to the vertex A, we have, by (6),

$$h = \frac{px}{2p-x} = -\frac{19{\cdot}67 \times 0{\cdot}955}{40{\cdot}295} = -0{\cdot}47 \text{ cm.}$$

Hence the optical system is equivalent to an ideal concave mirror of focal length 10·07 cm. or of radius 20·14 cm. placed with its reflecting surface 0·47 cm. *behind* the front surface of the glass mirror.

§ 7. *The micrometer method.* This method corresponds to the micrometer method of measuring the focal length of a lens system, a method recommended by Mr T. H. Blakesley*.

* *Geometrical Optics*, p. 91.

Let K (Fig. 3) be a point on the front surface of the mirror near the vertex A. Let FK cut the principal plane HZ in Z. Then, since the ray FZ passes through the focus F, the corresponding emergent ray is parallel to the axis and, since Z is its own image, this emergent ray passes through Z. Hence the emergent ray is ZP, where ZP is parallel to the axis AF. Since the incident ray FK passes through K, the emergent ray ZP passes through the image of K, i.e. the image of K lies somewhere on ZP. Hence, an object AK of height a in contact with the surface AK has its image of height b, where $b = HZ$. Since $CA = p$ and $CF = f$, we have $AF = p - f$, and thus

$$\frac{a}{b} = \frac{AF}{HF} = \frac{p-f}{f}.$$

Hence

$$f = \frac{bp}{a+b}. \quad \text{.....(7)}$$

In this method we are not concerned with the *position* of the image of K, but this image is at the point in which CK cuts ZP, since the incident ray CK becomes the emergent ray KC.

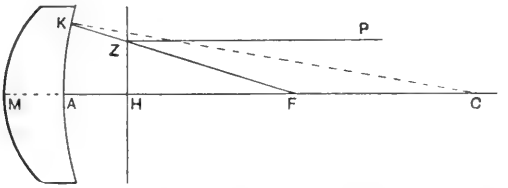


Fig. 3.

The measurements may be made by means of a microscope having a micrometer scale. The eyepiece is first adjusted so that the micrometer divisions are clearly seen. The two points A, K may be represented by two grains of lycopodium or by the vertices of two small *cleanly cut* triangles of tinfoil. If the surface of the mirror be breathed upon, the tinfoil will adhere to it. If the surface be *convex*, a glass scale may be used if it be placed with its divided face in contact with the vertex of the mirror system. Since the faces of the scale are parallel, the scale has no effect upon the number of micrometer divisions covered by either AK or its image. The microscope is then focused upon the two points so that there is no parallax between their images and the micrometer scale, and the reading of the micrometer scale is taken for each point. Let the difference of the readings be m. The microscope is then moved, without changing the distance between its object glass and eyepiece, so that it is focused upon the images of K and A, and the difference of micrometer scale

readings is again found; let this be n. Then, since $a/b = m/n$, we have, by (7),

$$f = \frac{np}{m+n}. \quad \text{.....(8)}$$

The distance $p = AC$ is found just as in § 5.

It is not necessary that the microscope should be provided with a sliding carriage, but, if this is the case, the difference of the scale readings of the carriage will give the distance x which is required in Gauss's method. If a glass scale is used to provide the points A and K, it will have no effect upon the value of x, since it displaces both K and its image through equal distances parallel to the axis.

§ 8. *Practical example.* The following results were obtained by G. F. C. Searle using a system formed by a double-convex lens placed in front of a plane mirror.

The lens was 1·2 cm. thick and was placed at about 2·8 cm. from the plane mirror which was formed of a piece of glass about 0·3 cm. thick, silvered on the back. The focal length of the lens was about 20 cm. The plane mirror was not of very good quality and thus very accurate readings were impossible. A glass scale was used to provide the points to be observed. In the table m and n correspond to 1 mm. on the glass scale.

Glass scale seen directly			Image of glass scale		
Scale readings for 7 mm.		$7m$	Scale readings for 4 mm.		$4n$
10·4	71·2	60·8	13·5	69·0	55·5
12·0	73·0	61·0	18·0	73·7	55·7
18·8	79·6	60·8	20·4	76·1	55·7
20·7	81·6	60·9	30·0	85·0	55·0

Mean value of $7m = 60{\cdot}88$. Mean value of $4n = 55{\cdot}48$.
Hence $m = 8{\cdot}70$, $n = 13{\cdot}87$.
Distance of centre C from vertex A, 20·16, 20·06, 20·06, 20·02 cm.
Hence $p = 20{\cdot}08$ cm.

Then, by (8), $$f = \frac{np}{m+n} = \frac{13{\cdot}87 \times 20{\cdot}08}{22{\cdot}57} = 12{\cdot}34 \text{ cm.}$$

By (6), $$h = p - 2f = 20{\cdot}08 - 24{\cdot}68 = -4{\cdot}60 \text{ cm.}$$

Hence the system is equivalent to an ideal concave mirror of focal length 12·34 cm. placed with its surface 4·60 cm. behind the front surface of the optical system.

The focal length was also measured by Gauss's method.
The mean value of x was found to be $-11{\cdot}61$ cm. Then, by (5),

$$f=\frac{p(p-x)}{2p-x}=\frac{20{\cdot}08\times 31{\cdot}69}{51{\cdot}77}=12{\cdot}29 \text{ cm.}$$

By (6), $$h=\frac{px}{2p-x}=\frac{20{\cdot}08\times(-11{\cdot}61)}{51{\cdot}77}=-4{\cdot}50 \text{ cm.}$$

§ 9. *The goniometer method.* This method is similar to the goniometer method for measuring the focal length of a lens system*. Let GF (Fig. 4) be an object of length s in the focal

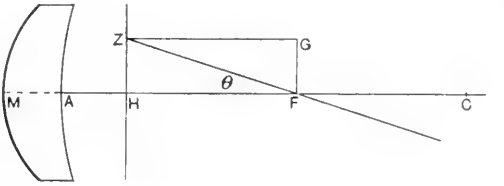


Fig. 4.

plane. Then, if GZ, drawn parallel to the axis, cuts the principal plane HZ in Z, the incident ray GZ will give rise to the emergent ray ZF. If the angle ZFH be θ, then, for small angles,

$$\theta = ZH/HF = s/f.$$

Or $$f = s/\theta. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(9)$$

Since G is in the focal plane, all the rays starting from G which emerge from the system are, on emergence, parallel to ZF; it is supposed, of course, that the aperture of the system is small enough to prevent aberration from being conspicuous. If these emergent rays are received by a goniometer, which has been focused for "infinity," an image of G will be formed on the cross-wire of the goniometer, if the arm of the goniometer is properly

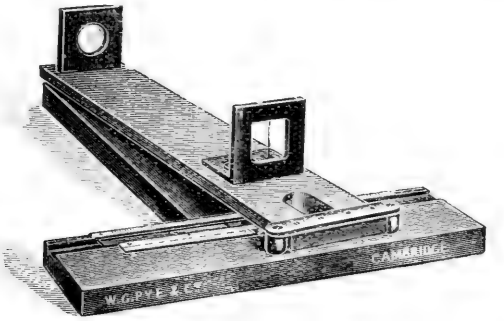


Fig. 5.

* G. F. C. Searle, *Proceedings of the Optical Convention*, Vol. II. 1912, p. 165.

directed. Thus, θ is the angle between the two positions of the arm in which (1) the image of F and (2) the image of G is focused on the cross-wire.

The goniometer devised by the author in conjunction with W. G. Pye and Co. is shown in Fig. 5*.

The apparatus is arranged as in Fig. 6. The mirror system AM is set up with its axis horizontal. A glass scale S is attached to a sliding carriage (Fig. 2), the direction of motion of the carriage being parallel to the axis of the mirror system. The *divided* face of the scale is turned *towards* the mirror and care is taken that the plane of this face is perpendicular to the axis of the system. The goniometer is then set with its axis coinciding with that of

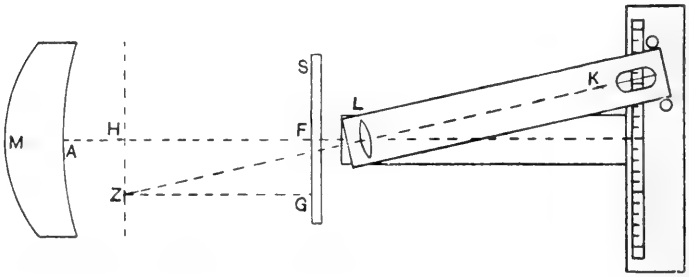


Fig. 6.

the system and the scale S is adjusted by aid of the sliding carriage so that the image of a nearly central dividing line is seen sharply focused (without parallax) upon the cross-wire of the goniometer. The end of the goniometer should be close to the scale S, care being taken that it is not so close that the arm comes into contact with the scale when it is turned about the pivot. The goniometer may, conveniently, be bolted to a table carried by a Compound Laboratory Stand (W. G. Pye and Co.). For accurate work the various pieces of apparatus must be *firmly* supported.

The distance AF between the vertex A and the scale S, which has been set so that its divided face passes through F, is measured by aid of an adjustable distance piece. Or the scale reading of the carriage is taken and then the carriage is adjusted so that a rod of definite length—10 or 20 cm.—touches A and F. From the two scale readings and the length of the rod, the distance AF is computed.

The goniometer arm is then turned so that the cross-wire K is brought to coincidence with the images of a number of dividing

* The instrument is described in my paper "Experiment on the harmonic motion of a rigid body" (*Cambridge Phil. Soc. Proc.* Vol. XVIII. p. 31) and in the manual on *Experimental Harmonic Motion* which it is hoped will be published in 1915.

lines on S in turn, the interval between successive lines being constant, and the scale reading of the arm is taken in each case. From these readings the mean distance, d cm., on the goniometer scale corresponding to s cm. on the glass scale is found. The distance, l cm., from the centre of the pivot of the goniometer to the edge of the goniometer scale is also measured. Then, if ϕ be the angle corresponding to s cm. on the glass scale, $\phi = d/l$ radians, for small angles. Then by (9), since $\theta = \phi$,

$$f = s/\theta = s/\phi = sl/d \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

The thickness of the scale S has no influence on the result. The rays emerging from the mirror system pass through S, but their *directions* are unchanged by the passage.

§ 10. *Practical example.* The following results were obtained by G. F. C. Searle using the same mirror system as was used by Mr Heath in § 6.

Length of goniometer arm, i.e. distance from centre of pivot to edge of scale $= l = 40{\cdot}00$ cm.

When the glass scale was set in the focal plane, with its divided face turned towards the mirror system, the following readings were obtained:

Reading of glass scale	Reading of goniometer scale	Reading of glass scale	Reading of goniometer scale	Change of goniometer reading for 1 cm. on glass scale
4·0 cm.	6·58 cm.	5·0 cm.	10·59 cm.	4·01
4·5	8·59	5·5	12·59	4·00
5·0	10·59	6·0	14·60	4·01

Mean 4·007 cm.

Two other sets of observations, each with a fresh setting of the glass scale, gave the mean values 4·010 and 4·030. Taking the general mean, $d = 4{\cdot}016$ cm.

Hence, by (10), since $s = 1$,

$$f = sl/d = 40/4{\cdot}016 = 9{\cdot}96 \text{ cm.}$$

Distance AF of glass scale from vertex $A = f + h = 9{\cdot}59$ cm.

Hence $$h = 9{\cdot}59 - 9{\cdot}96 = -0{\cdot}37 \text{ cm.}$$

The values found by Mr Heath were $f = 10{\cdot}07$, $h = -0{\cdot}47$ cm.

§ 11. *A special system.* An interesting system may be formed by a convex lens and a plane mirror, the distance between the lens and mirror being greater than the focal length of the lens; the distance may be conveniently about twice the focal length. With this system the two real self-conjugate points, viz. the "centre" and

the principal point, are both in front of the lens. The focus of the lens remote from the mirror is one point, and here an inverted image is obtained. This point is therefore the "centre" of the system. The other point is that point which has for its conjugate with respect to the lens the point where the axis of the system cuts the plane mirror. Here the image is erect. This point is therefore the principal point*. But, since the image is erect, it is *hidden* by the object when the adjustment is complete, and this furnishes a test of the adjustment. If a well-polished needle be used and a light from a flame be concentrated upon it by means of a large lens, a halo of coloured light will be seen round the pin, the colour being due to chromatic aberration and changing with the position of the pin. The light should fall as much as possible on the back of the needle.

Since the "centre" of the system is *nearer* to the lens than the principal plane of the system, the system behaves like an ideal *convex* mirror.

* Mr R. E. Baynes has pointed out to me that the position of the principal plane of this system, or of any more general system, is independent of the curvature of the surface M (Fig. 1), at which the reflexion occurs, provided that the vertex of that surface is always in the same position.

On the Electrification given to the Air by a Steam Jet. By W. A. Douglas Rudge, M.A., St John's College.

[*Received* 30 March 1915]

Many investigators from the days of Faraday and Armstrong have made observations on the electrification manifested during the escape of steam under high pressure and it is well known that an insulated body suitably placed in a current of steam does acquire a charge. Armstrong showed that the magnitude of the charge increased enormously with increase of pressure, and devised his Hydro-electric machine by which large quantities of electricity were produced. The sign of the charge given to conductors placed in the current of steam was usually positive but occasionally negative, and the magnitude of the charge seemed to be associated with the amount of disturbance present in the jet, for a quiet flow of steam was accompanied by a small charge and a noisy flow by a large one.

The writer has shown* that when steam under pressure escapes, it gives to the air a strong charge which persists for some time after the condensation of the steam. The experiments now to be described were made with the object of ascertaining the conditions under which a positive or a negative charge was given to the air, and also to find out whether the jet was uniformly charged.

The apparatus employed consisted of a small spherical copper boiler 20 cm. diameter furnished with a pressure gauge to indicate pressures up to about one and a half atmospheres. A set of taps with nozzles of different sizes and materials allowed of the flow of steam being controlled. The jet could be projected in any direction, but the actual direction did not make an appreciable difference in the sign or magnitude of the charge when the observation was made in a room, but out of doors a vertical jet seemed to give the greatest charge. The boiler was heated by a gas burner or by an electrical arrangement, the latter being particularly useful when inflammable liquids were mixed with the water. The charge carried by the steam could be easily detected by arranging an insulated conductor in the path of the steam, whilst that given up to the air was indicated by a small radium coated collecting plate attached to an electroscope.

The electroscope used was of the usual type with single gold leaf but was made double, and the images of the gold leaves were projected upon scales fixed to the wall a metre or two away. The movements of the leaves could thus readily be followed even when

* *Proc. Roy. Soc.* A. Vol. 90, 1914.

manipulating the apparatus and the charges upon the steam and the air simultaneously observed.

When the steam was allowed to escape from the boiler, if the pressure was more than 150 mm. above that of the atmosphere, the steam, and the air also, was found to be strongly charged. An insulated conductor placed in the steam showed a positive charge, and the radium coated plate, which might be anywhere in the room, also indicated a positive charge. If the quantity of steam escaping was large the charges sent the gold leaves out at right angles to the supporting plates. By moving the radium coated plate about the room the charge could be detected in all parts, but the maximum effect was found at the centre, even though the boiler was at one end. That the air itself was actually charged was shown by carrying a large box which had been open in the room for some time, outside, and on putting an electroscope with a radium coated wire inside the box, a positive charge was found to be present. If the electroscope was placed upon the window-sill outside the room no charge was noticed, but on opening the window, and opening the door suddenly, a current of air could be sent out of the window, and that this carried a charge was at once shown by the electroscope, and this was the case some time after condensation of the steam.

In a closed room the charge given to the air by a very small quantity of steam will persist for a long time. In one particular case the boiler was arranged outside a room measuring about $5 \times 4 \times 3$ metres, and a jet of steam from an orifice 1 mm. in diameter sent into the room for about five minutes. This charged the air to such an extent that the gold leaf of the electroscope was sent out nearly at right angles, and the charge could be detected in the room for *more than one hour afterwards.* This same persistence of the charge after condensation of the steam has also been noted on a railway journey. If an electroscope having a collector attached is carried in a closed railway carriage, no charge is indicated. If the window is opened a charge can be detected upon that side of the train from which the steam is being carried. On entering a tunnel, a quantity of steam gets carried into the carriage, and at once a strong charge can be detected. If the window is now shut the charge will remain, notwithstanding the condensation of the steam, and may be noted many minutes afterwards.

A set of experiments was made to ascertain whether the presence of dissolved matter in the water had any influence on the charge given to the air by the steam. In most cases the charge was positive. Table I gives a few of the effects obtained. Only a few such are given but they are typical of the rest. It may be noted that volatile salts may give negative charges, as also did

other volatile bodies, but the charge was by no means invariable and generally changed to positive after the material had been volatilised.

TABLE I.

Liquid in boiler	Charge given to air	Remarks
Distilled water	Positive	Charge strong
Solution of salt	,,	,, ,,
,, caustic soda	,,	,, ,,
,, potassium nitrate...	,,	,, ,,
,, magnesium sulphate	,,	,, ,,
,, ammonium carbonate	Negative	After boiling for some time became positive
,, benzoic acid	Positive	
,, vinegar	Negative	Charge weak
,, hydrochloric acid ...	,,	Fairly strong
,, hydriodic acid	,,	,, ,,
Distilled water with a small piece of phosphorus.....	,,	Very strong at first
Iodine in potassium iodide	,,	Charge weak positive after a short time

It was not easy to prevent a charge being given to the air. If there was any visible condensation and the pressure was above one and a half atmospheres a charge was invariably obtained. Passing the steam through a red-hot tube had no effect in reducing the charge, but placing a large bunsen burner directly in front of the jet caused most of the charge to disappear if at the same time visible condensation ceased.

If a jet of steam gives a positive charge to the air it is obvious that a negative charge must be somewhere and one naturally looks for this on the boiler. This negative charge is readily detected by heating the boiler until a good pressure is obtained, and then placing it on an ebonite support. On connecting it to an electroscope the presence of the negative charge is at once shown.

As is well known, an insulated conductor held in a current of high pressure steam, acquires a positive charge, but under some circumstances a negative one can be obtained, and some casual observations appeared to indicate that the nature of the charge depended upon the position of the electrode. It seemed of interest to investigate this point and it was then found that a positive or negative charge could be obtained as desired, by suitably placing the exploring electrode. An arrangement was therefore made by which an exploring electrode could be moved about in the steam and the position noted where positive or negative charges were found.

The exploring electrode was made in many forms such as small cylinders, discs of metal or wire gauze, or plain wires; a fine needle seemed however to be the most satisfactory form to use. This needle was attached to a rod of ebonite and mounted upon the carriage of a travelling microscope, so that it might be moved in two directions at right angles and its position noted. The arrangement is shown in fig. 1. The tube projected about 20 cm. from the boiler, the orifice from which the steam escaped was 1 mm. in diameter. The electroscope E_1 was connected to the needle and served to indicate the charge in the steam jet; whilst E_2 gave the charge at the same time upon the air.

In starting an observation the needle was arranged at some distance from the end of the tube and the charges on the electroscopes noted. By moving the screw the needle could be moved nearer or farther from the orifice from which the steam was escaping and any variations in the charge observed. If the

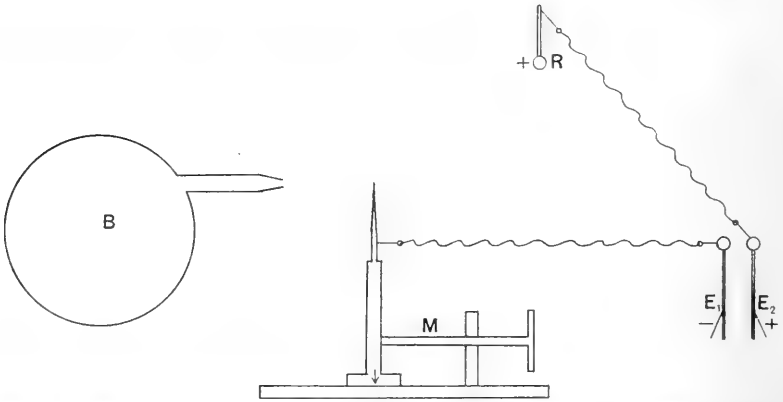


Fig. 1. The steam escapes from the small orifice in the tube attached to the boiler. The charge upon the air is indicated by the radium collector R, and electroscope E_2, and that upon the point by the electroscope E_1. E_1 and E_2 are at some distance, 2—3 metres, from the boiler.

distance of the needle was a few centimetres from the orifice from which the steam was escaping a positive charge was shown by E_1 and also by E_2 but on gradually lessening the distance by turning the microscope screw the charge shown by E_1 diminished in strength, and then changed sign, whilst that shown by E_2 remained always positive. The point at which the inversion of sign occurred was fairly sharply defined, as is shown in Table II. The results there shown were obtained with an orifice 1 mm. in diameter, and a pressure of steam about 60 cm. above that of the atmosphere.

TABLE II.

Distance of electrode from orifice	Charge indicated by electrode
15 cm.	+
10 „	+
5 „	+
2 „	+
1·9 „	0
1·5 „	−
2·0 „	+
1·7 „	−
1·5 „	−
1·8 „	0
1·9 „	+
3·0 „	+
2·0 „	+
1·7 „	−

The position of the point of inversion varied with the pressure, and the greater the pressure the farther the point was from the orifice, but there did not seem to be any very definite relation between the pressure and the position of the point, owing probably to irregular condensation.

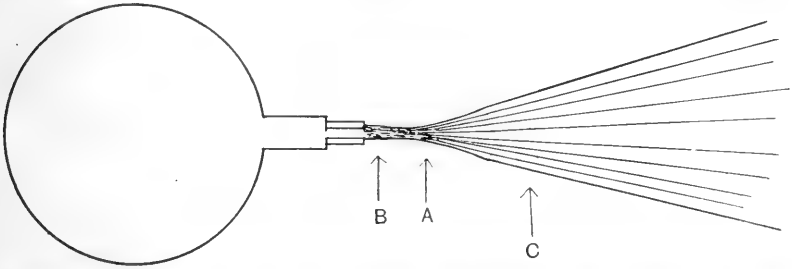


Fig. 2. The steam escaping from the boiler forms a cone somewhat as shown. At points in the cloud above *C* a *positive* charge would be indicated; at *A* *no charge*, at *B* a *negative* charge.

If a jet of steam escaping under pressure is closely examined it may be seen that visible condensation does not occur until the steam has gone some distance from the orifice, that is the steam remains 'dry' and the jet of steam takes the form of a short cylinder, which opens out into a cone at some distance, which depends upon the diameter of the orifice and upon the pressure of the steam (fig. 2). The change of sign seems to occur at the point where the cylinder opens out into a cone, that is where the steam passes from the dry to the wet condition. The outer layer of the

dry steam cylinder is also apparently positive but not very strongly so. A small insulated-ring shaped electrode was arranged to surround the jet at less than 1 cm. from the orifice. The ring could be moved so as to cut through the jet of steam, and an electroscope joined to it indicated the charge acquired by the ring in different positions. Fig. 3 shows the charges obtained in different positions (*a*), (*b*), (*c*). No charge was indicated when the ring was at (*a*), a positive charge when the ring just grazed the outside of the steam jet, and a strong negative one when the wire was at the centre of the jet. It must be remarked that the presence of the wire in the jet rather disturbs it, and of course a part of the wire is passing through the outer positive layer, but the negative charge in the central part of the steam column is evidently much greater than the positive charge on the outer portions.

The experiments were repeated in the Engineering Laboratory at Cambridge, using much larger volumes of steam and much

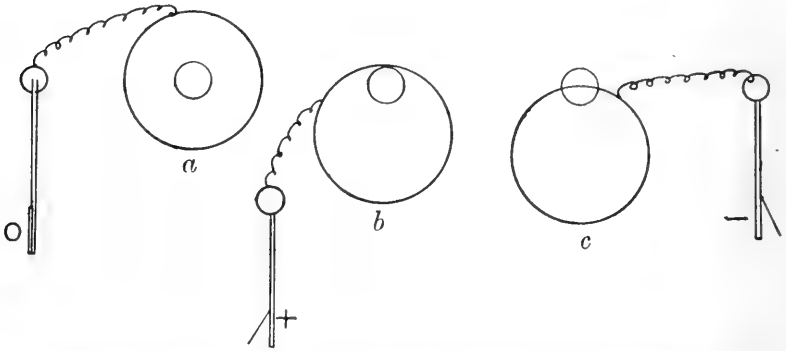


Fig. 3. When the wire ring is in the position shown at *A* no charge is indicated by the electroscope, at *B* a positive charge is shown and at *C* a negative one.

higher pressures than could be obtained with the small boiler, but, save in the magnitude of the charges obtained, the general results were the same. Pressures up to eight atmospheres were used and jets varying from 1 mm. up to 8 mm. diameter were employed. The charge given to the exploring electrode was the same as with the smaller boiler, but the inversion point was much farther away from the orifice, in one case nearly 5 cms. The air of the boiler house was very strongly charged with positive electricity.

When a clean needle electrode was placed in the current of steam it was observed that the steam condensed upon the point in the form of small drops which quickly coalesced into one large one. This large drop was fairly permanent when it had reached a certain size and the rate of growth and evaporation balanced. It then clung tenaciously to the needle, and when the charge shown by

the needle was *positive*, the drop was on the side of the needle *nearest* to the orifice, and when *negative*, on the side *farther* from the orifice. These drops were photographed and are shown in fig. 4; the position is so definite that by noting where the

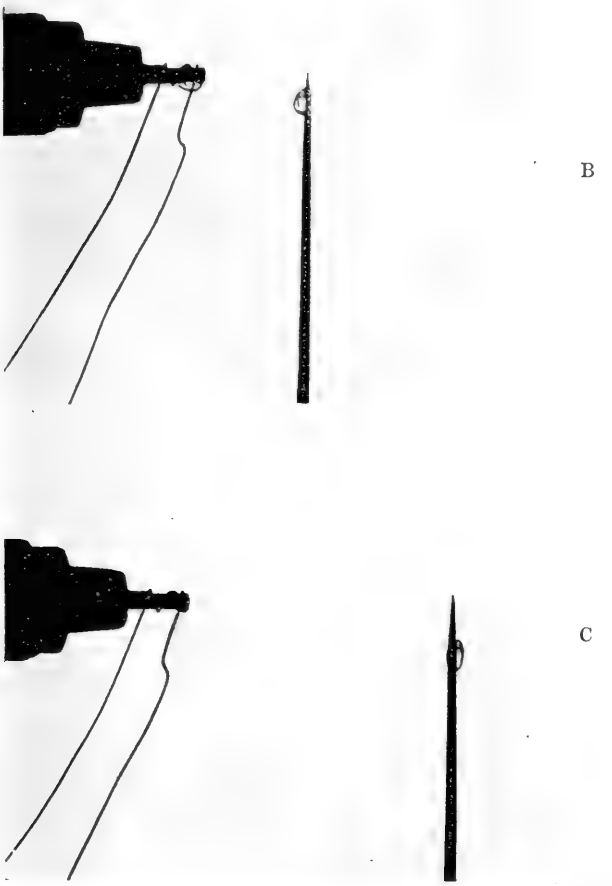


Fig. 4. The needle has a negative charge when the drop is *towards* the jet, and a positive charge when *away* from the jet, the positions with reference to the jet being shown at *B* and *C*, fig. 2.

drop is seen, the charge upon the needle may be inferred. The cause of the position assumed by the drop is probably in some way related to the eddy currents present in the steam after striking an obstacle.

Origin of the charge given to the air.

The experiments do not throw much light on the origin of the charge given to the air, but two or three possible causes suggest themselves as being at least contributory.

(1) The charge might be thought to be due to the condensed water being atomised by the outrushing steam, but in that case, reasoning by analogy with the electrification near a waterfall, it should be negative, whilst it is actually positive.

(2) Friction of the escaping jet at the orifice. Undoubtedly some charge originates here if the steam passing through the orifice is not dry, but this will account for only a part of the charge, for the interposition of a fine needle in the path of the steam will considerably increase the charge given to the air, and at the same time the colour of the steam cloud changes, the strongest charges being produced when the colour appears blue; even a partial stoppage of the orifice which causes the cloud to become blue is accompanied by an increase of the charge.

(3) The charge may be due to the friction of the drops of condensed water against each other, much in the same fashion as is the charge produced during the raising of a dust-cloud*. This suggestion is put forward tentatively, but the fact that the finer the water particles—as shown by the colour of the cloud—the stronger the charge, seems to substantiate it, for here by analogy the finer the particles of dust the greater is the charge given to the air.

Summary.

(1) Some experiments have been made with steam jets which show that a positive charge is given to the air by the action of the jet, and this charge persists for some time after the condensation of the steam.

(2) The charge given to the air by the steam from pure water is always positive, and the presence of non-volatile substances dissolved in the water has no influence on the charge.

(3) Volatile bodies mixed with the water may give a negative charge to the air.

(4) The charge acquired by the air appears to be due to something which occurs in the cloud of condensed vapour.

(5) Positive and negative charges can be obtained simultaneously from the same jet, by suitably placing exploring electrodes in the path of the steam.

My thanks are due to Professor Sir J. J. Thomson for the use of the Cavendish Laboratory where the greater part of the work was carried out.

* *Roy. Soc. Proc.* A. Vol. 90, 1914.

Note on Dr Searle's experiment on the harmonic motion of a rigid body. (Proceedings of the Cambridge Philosophical Society, November 24, 1913.) By Sir GEORGE GREENHILL.

[*Received* 20 February 1914.]

The experiment provides an instructive exercise in measurement for the student of mechanical physics, and the practical details are useful for anyone who wishes to repeat it.

But as the experiment is carried out in a gravity field, and the calibration of the wire torsion is made (fig. 2) by weights in a scale pan, it would be more genuine and instructive, in my opinion, if the torsion couple $\mu\theta$ was recorded as it is measured, in gram-centimetres.

This makes $T = 2\pi\sqrt{\dfrac{K}{g\mu}}$ in formula (1), and then $\dfrac{K}{\mu}$ is the length of the equivalent simple pendulum, 44·42 cm in the experiment, with

$$K = 10^4 \times 9{\cdot}894 \text{ g-cm}^2, \quad \text{and} \quad \mu = \frac{3{\cdot}18}{10^{-3} \times 1{\cdot}428} = 10^3 \times 2{\cdot}227 \text{ g-cm}$$

per radian; and when this pendulum length is measured, beating time alongside with the torsional vibration, like a metronome but with invisible oscillation, the experiment is complete.

No need for a clock or a watch, in this the method of Galileo.

And the factor g, required for the conversion into absolute measure of force, cannot be measured directly by experiment with any accuracy approaching the determination of the length L of the pendulum which beats the second; so that g is always calculated indirectly through the formula

$$g = \pi^2 L.$$

The formula for the beat t of a pendulum of length l is thus more properly

$$t = \sqrt{\frac{l}{L}}, \text{ instead of } \pi\sqrt{\frac{l}{g}}.$$

Thus if we take the length as 25 cm, in round numbers, of the half second pendulum for which $T = 1$, then $T = 1{\cdot}337$ would have an equivalent pendulum length

$$l = 25 \times (1{\cdot}337)^2 = \left(\frac{13{\cdot}37}{2}\right)^2 = (6{\cdot}685)^2 = 44{\cdot}69 \text{ cm.}$$

But with $l = 44{\cdot}42$ cm, the length of the half second pendulum would be $44{\cdot}42 \div (1{\cdot}337)^2 = 24{\cdot}85$ cm, making the length 99·4 cm of the pendulum which beats the second.

On the assumption of constant μ, and neglecting air drag, the torsional period is the same for visible and invisible oscillation.

But the circular correction is required for the equivalent gravity pendulum. If its invisible oscillation is in tune with the torsion, then swinging through an angle of $D°$ it would lose one beat in $N = \left(\frac{8 \times 57{\cdot}3}{D}\right)^2$ beats, observable by the Method of Coincidences, described in Maxwell's *Matter and Motion*, p. 106; thus $D = 7°.64$ makes $N = 3600$, one second beat in the hour.

Or if the pendulum synchronizes with the torsional oscillation when it swings through $D°$, its equivalent length must be stretched $200\left(\frac{D}{8 \times 57{\cdot}3}\right)^2$ per cent, to give the equivalent length for invisible oscillation; this is $\frac{1}{18}$ °/₀ for $D = 7°.64$.

Air drag and buoyancy can then be measured incidentally: and here is an opportunity of showing the Bessel pendulum at work, made of a length, say half a metre, of bicycle tube, filled with lead at one end. The theory is given in my *Notes on Dynamics*, p. 183.

In a clock pendulum with a cylindrical bob, the air correction is about $\frac{1}{40}$ °/₀ of its length l, and so would be felt in the next recorded figure, the fifth, of the experiment.

To fasten a thin wire firmly in a supporting plate of copper, Mr C. V. Boys found the method useful described in *La Science amusante, par Tom Tit*, where a needle enclosed in a cork is driven through copper to punch a hole, and the wire is then clinched in the hole and the copper hammered to hold it tight.

With the copper plate screwed on to the end of a bracket, held by Six Point Contact with supports fixed firmly in the wall, the only defect of rigidity is in the elastic flinch of the bracket.

I should like to put in a plea here for Dr Schuster's name *Simple Vibration*, instead of Harmonic Vibration, the word harmonic being reserved for the overtones in a Fourier Series.

Further Experimental Researches on Insect Flagellates introduced into Vertebrates. By H. B. Fantham, D.Sc. Lond., M.A., Christ's College, Cambridge, and Liverpool School of Tropical Medicine, and Annie Porter, D.Sc. Lond., F.L.S., Beit Memorial Research Fellow, Quick Laboratory, Cambridge.

[*Read* 10 May 1915.]

Contents.

I. *Introduction.*

The experiments summarised in this paper were made together with and in continuation of those presented to the Society on November 23 last, and published in this volume, pp. 39—50. They record the results of the experimental introduction into vertebrates of flagellates belonging to the genera *Herpetomonas* and *Crithidia*, which live naturally in the digestive tracts of various insects. The range of experiment has been extended to include examples from most of the great classes of the Vertebrata. The work is necessarily prolonged and tedious, as some of the animals used live for months, and consequently relatively few experimental vertebrates can be managed and observed at any one time. We commend this remark to the would-be critic, who may say that enough experiments have not been performed; we would also remind him that destructive criticism is very common in these days, but of little value *per se*, unless accompanied by a constructive policy. Indeed, after the important and suggestive researches of Rogers in 1904—05 on the cultural herpetomonad forms of *Leishmania donovani*, and the work of Patton in 1907 on the complete life-cycle of a *Herpetomonas*, it is little short of amazing that experiments of the nature described in this paper were not immediately undertaken in endemic areas of Kala-azar. In other words, little regard was given primarily to the possible

invertebrate hosts of *Leishmania*, while much attention was concentrated on attempts to infect the usual laboratory mammals with the parasite or with cultures of it. It seems, then, that one important aspect of the subject of leishmaniasis unfortunately has been overlooked for a long time.

II. *Material and Methods.*

The flagellates used in this research include *Herpetomonas jaculum* (Léger) parasitic in the gut of the Hemipteran, *Nepa cinerea*; *H. stratiomyiae* (Fantham and Porter) from the digestive tract of the larva of the Dipteran, *Stratiomyia chameleon*; *H. pediculi* (Fantham) from the gut of the body-louse, *Pediculus vestimenti*; and *Crithidia gerridis* (Patton) from the alimentary tract of the Hemipteran, *Gerris paludum*. It will be seen that some of these insect hosts are blood-suckers, while others are not.

The vertebrate hosts included sticklebacks (*Gasterosteus aculeatus*) among the Pisces; newts (*Molge vulgaris*), frogs (*Rana temporaria*) and toads (*Bufo vulgaris*) among the Amphibia; lizards (*Lacerta vivipara*) and a grass-snake (*Tropidonotus natrix*) among the Reptilia, and mice (*Mus musculus*) among the Mammals.

The insect flagellates were introduced into the vertebrates by inoculation or by feeding. No ectoparasites and no haematozoa were present on or in the vertebrate hosts at the commencement of the researches.

Blood films were made from time to time during the life of the experimental animals, while smears of the internal organs were prepared at autopsy. Some of the preparations were fixed while moist with osmic vapour followed by absolute alcohol, while others were fixed wet with Bouin's fluid. The stains used were Giemsa's solution, haematoxylin and eosin, and occasionally iron-haematein.

Control vertebrates were kept in each case. They remained healthy and unparasitised, and lived longer than the experimental animals.

III. *Experimental Work.*

As experiments with mammals were detailed in our former paper, it may be of interest to begin with new experiments on cold-blooded hosts and then to take further experiments with mammals, involving the use of flagellates hitherto untried.

(*a*) *Experiments with Pisces.*

Experiment 1 (A.P.). A male stickleback, *Gasterosteus aculeatus*, was inoculated subcutaneously with the mid-gut of a *Nepa cinerea* containing flagellate forms of *Herpetomonas jaculum*. The

next day a slight swelling appeared at the site of inoculation. This soon disappeared. The blood examination was negative, as it was on subsequent days. Six days after inoculation, the fish died. Smears of the tissue near the site of inoculation contained contracting parasites approaching a non-flagellate condition, and also flagellate forms. Elongate and flagellate herpetomonads were present in the heart blood, while leishmaniform and elongating parasites were present in the liver and spleen. The control fish died a day after the experimental one, but no herpetomonad in any stage of development was found in its organs.

Experiment 2 (H.B.F.). A male stickleback was fed with the gut of a *Nepa cinerea* containing a few flagellate forms of *Herpetomonas jaculum.* It only lived two days, and no parasites were found at autopsy.

(*b*) *Experiments with Amphibia.*

Experiment 3 (A.P.). A large male frog, *Rana temporaria,* was inoculated intraperitoneally with the gut contents of three *Gerris paludum,* which contained flagellate and a few postflagellate forms of *Crithidia gerridis.* Two days after inoculation, some oval and elongating parasites were found in the blood. Subsequent examinations were negative. The frog became thinner and died on the evening of the 29th day after the experiment began. Smears of the organs showed that the liver contained a number of the oval, encysted, postflagellate forms of the protozoon, as well as fully developed flagellates presenting the typical crithidial facies. Elongating forms, transitional between the non-flagellate and full flagellate forms, occurred both free and in mononuclear cells in the liver. A few cells contained two rounded or elongating Crithidia.

It may be mentioned that here, as in all our experiments in which *Crithidia gerridis* was used, no transition to a trypanosome was ever seen.

Experiment 4 (H.B.F.). A male frog, *Rana temporaria,* was inoculated intraperitoneally with the fore-gut of a *Nepa cinerea,* containing preflagellate and young flagellate forms of *Herpetomonas jaculum.* No exudate appeared at the site of inoculation. On the 4th day after the commencement of the experiment, a leishmaniform parasite was found in an endothelial cell. Blood smears were negative from then to the 10th day, when leishmaniform stages were found in the blood. Certain of these forms showed a thin cyst wall, and certainly represented the postflagellate form as produced at the end of the developmental cycle in the insect host. Further blood examinations were negative until the 31st day, when both leishmaniform and elongating herpetomonads were present. Subsequent blood examinations

were negative. The frog died on the 54th day after inoculation. At autopsy the animal appeared somewhat anaemic, and both flagellate and postflagellate herpetomonads were present in the liver.

Experiment 5 (A.P.). A large male toad, *Bufo vulgaris*, was inoculated subcutaneously with the rectal contents of one *Nepa cinerea*, containing recently formed postflagellate stages of *Herpetomonas jaculum*. A slight swelling and reddening at the site of inoculation occurred on the second day, but disappeared on the third. Blood smears were taken daily. Six days after inoculation, a leishmaniform parasite was found in the blood, but blood smears on subsequent dates were negative, except on the eleventh day, when leishmaniform and young flagellate parasites were seen. The toad died forty days after inoculation. At the autopsy, leishmaniform parasites were found in the liver.

Experiment 6 (H.B.F.). A mature female toad, *Bufo vulgaris*, was inoculated intraperitoneally with the gut contents of one *Nepa cinerea*, containing young flagellate forms of *Herpetomonas jaculum*. No untoward symptom of any sort was noticed in this toad. Blood smears were made daily for a period of forty-five days, but all were negative. Further periodic examinations were made and were negative. The experimental toad and its control animal were killed at the end of eighty days. The internal organs of both toads showed no herpetomonad parasites.

Experiment 7 (A.P.). A young male newt, *Molge vulgaris*, was fed with the mid-gut of a *Nepa cinerea*, containing a few flagellate forms of *Herpetomonas jaculum*. The newt became less active but otherwise showed little signs of illness. Blood examinations were made daily for six days, but no parasites were found therein. Unfortunately, the newt escaped in the laboratory, and when found three days later, was dead and too decomposed to allow of detection of parasites. From the examination of the blood alone, there does not appear to have been an infection.

Experiments with tadpoles and *Crithidia gerridis* are in progress.

(*c*) *Experiments with Reptilia.*

Experiment 8 (H.B.F.). A male lizard, *Lacerta vivipara*, was fed on the gut contents of two *Gerris paludum*, one containing preflagellates and the other mostly postflagellate forms of *Crithidia gerridis*. The number of parasites in the feeding material was small. Blood examinations were made. Five days after the infective feed, the blood was found to contain rounded leishmaniform elements, some in process of division. Dividing parasites occurred both free and within leucocytes. Similar intracellular and free forms were present in the blood

also on the 6th day, and leishmaniform elements on the 18th day after inoculation. Nineteen days after the commencement of the experiment, the lizard died. During the last week of its life, it did not feed so well as its control. At autopsy the organs appeared to be normal, except for a slight softening of the liver. Preparations, however, showed that the liver, heart and bone-marrow contained leishmaniform and elongating parasites, while in the spleen, fully formed flagellates, presenting the typical Crithidia facies, occurred. Smears of the lung and kidney were negative. The presence of dividing forms showed conclusively that *Crithidia gerridis* can not only maintain itself, but can also perpetuate its race in a vertebrate host.

Experiment 9 (H.B.F.). A second male lizard was fed with teased portions of the liver of the first one, which was infected with *Crithidia gerridis*. After four days the lizard refused to feed and died on the sixth day. At autopsy, parasites were found in the internal organs, their appearance and distribution being like those of the "seed" animal.

Experiment 10 (H.B.F.). A third female lizard has been inoculated intraperitoneally with heart blood from the preceding animal, and has already shown a few leishmaniform parasites. Three passages of the parasite have thus been accomplished.

Experiment 11 (A.P.). A male grass-snake, *Tropidonotus natrix*, was fed with the guts of four *Nepa cinerea*, poorly infected with *Herpetomonas jaculum*, a few flagellates and preflagellates only being present. Blood examinations were made daily. On the 4th, 10th, 13th and 19th days after feeding the blood showed oval, leishmaniform parasites, mostly free. On the 7th and 15th days non-flagellate dividing forms were present, as well as leishmaniform bodies. Elongating parasites were seen on the 11th day. On the 8th day after feeding a single erythrocyte was found containing a small uninucleate parasite, perhaps corresponding to the forms described by Laveran and Franchini.

The snake died 20 days after the experiment began. Smears of the organs were made. The liver contained well formed herpetomonad flagellates and some leishmaniform parasites. The spleen harboured a few small flagellates. Dividing non-flagellate forms, as well as ovoid parasites, were present in the kidney. Leishmaniform bodies were found in the lungs. Smears of the spinal cord near the middle of the body showed the presence of non-flagellate forms, both ovoid and in process of division.

(*d*) *Experiments with Mammalia.*

Experiment 12 (H.B.F.). A young male mouse, *Mus musculus*, weighing 4·9 grams, was fed with the rectal end of the gut of a larva of *Stratiomyia chameleon*. The gut contained

a few flagellates and some postflagellates of *Herpetomonas stratiomyiae*. Twenty-four hours after the infective feed, the mouse commenced to shiver. The shivering became violent and continued so until the animal died. Attacks of rigor occurred early each morning but passed away after a couple of hours. The mouse became emaciated and died five days after the infective feed.

Examination of the blood of the mouse was made daily, but no parasites were found therein. Smears of the organs made at autopsy showed that infection had occurred. Leishmaniform elements, mostly free, occurred in the liver, lung and kidney, the liver showing the greatest number of parasites. Some of the herpetomonads in the liver were in the multiplicative phase, showing division of both nucleus and blepharoplast, as well as the commencement of cytoplasmic cleavage. Other parasites showed an indication of encystment, forming postflagellates like those found in the faeces of insects. Occasionally, seemingly uninucleate elements were found, but on careful analyses, these were ascertained to be really binucleate, the blepharoplast being superimposed upon the nucleus. Fully developed flagellates were not found, but some in process of elongation were present.

Experiment 13 (A.P.). An adult female mouse was fed with a number of body lice, *Pediculus vestimenti*, obtained from several sources. Some of these lice contained *Herpetomonas pediculi*. No untoward symptoms were noticed for some time, but during the latter part of its life, the mouse became emaciated and was subject to attacks of shivering. It lived for 72 days. Blood smears were made daily. Oval, leishmaniform bodies were found on the 4th, 7th, 11th, 19th, 24th, 29th, 32nd, 39th, 47th, 56th and 69th days after feeding on the lice. Dividing forms were noticed on the 26th, 35th and 62nd days, elongating parasites on the 29th, 37th, 42nd, 58th and 67th days, while mature flagellate herpetomonads were seen on the 30th, 44th, 64th and 71st days after feeding. At autopsy, the body was found to be emaciated and the mouse weighed 2 grams less than its control, both being of equal weight, 11·7 grams, at the commencement of the experiment. The liver was pale and the spleen enlarged and brittle. Oval, leishmaniform parasites were present in the liver, spleen, bone-marrow and heart blood. Elongating forms and a flagellate herpetomonad also occurred in the liver.

Experiment 14 (A.P.). An adult female mouse was fed with part of the liver of the above-mentioned mouse, infected with *Herpetomonas pediculi*. It soon began to lose weight and became very weak. It died on the 15th day. At autopsy, leishmaniform and flagellate *H. pediculi* were found in the organs, much as in the mouse from which it had been infected. The parasite is thus transmissible from one mammal to another.

An attempt at culture of heart-blood of this mouse on blood-agar met with very slight success. Further experiments are in progress.

Experiment 15 (H.B.F.). An adult female mouse, weight 12·5 grams, was inoculated intraperitoneally with the gut contents of two *Gerris paludum* infected with *Crithidia gerridis*, flagellates chiefly being present. The inoculation material did not contain a large number of flagellates. A small sore developed at the site of inoculation but rapidly healed. The hair around the inoculation point came away and a bald patch was thus formed. The mouse became thinner than its control. It was killed in extremis on the 40th day after inoculation. Its weight was then 10·8 grams. Blood smears were examined daily. On the 3rd, 4th, 5th, 9th, 12th, 15th and 20th days after inoculation, oval, non-flagellate parasites were observed in the blood. On the 5th day, dividing organisms were seen. Intermediate elongating forms were observed on the 5th and 8th days, while on the 13th and 20th days, fully formed flagellate Crithidia were present in the blood. The smears of organs when examined showed oval non-flagellate forms in the heart, liver, spleen and lungs. Elongating parasites occurred more especially in the liver, and a few small flagellates were found in the liver and heart.

Experiment 16 (A.P.). An adult female mouse, weight 14 grams, was inoculated subcutaneously with the gut of a *Gerris paludum*, containing some preflagellates and a few flagellate forms of *Crithidia gerridis*. Blood films were taken daily for a period of over two months, but all were negative. The mouse was then killed, but at autopsy no parasites were seen in any of its organs. It had increased in weight by 1·2 grams.

Experiment 17 (H.B.F.). An adult male mouse was fed with the gut of a *Gerris paludum* containing flagellate and some post-flagellate forms of *Crithidia gerridis*. Blood smears were made daily. On the day after feeding, a small, non-flagellate parasite was seen in the blood. Similar non-flagellate forms were found on the 8th, 11th, 13th, 20th and 30th day after the infective feed. Dividing, oval organisms were observed on the 4th and 20th days. Crithidia in various stages of elongation occurred in the circulating blood on the 7th, 17th, 20th and 26th days, and fully formed flagellates showing an undulating membrane and flagellum were present on the 12th and 35th days. The mouse died on the 38th day after the infective feed. It had shown weakness, with a staggering gait, for some days previously. It weighed 10·75 grams at death, the control mouse from the same litter weighing 12·7 grams. The liver was pale, the spleen brittle and slightly enlarged. The liver contained a few non-flagellate parasites, and some uninucleate forms were also present, these latter being rare.

Similar parasites occurred in the heart. The spleen contained some small, oval, non-flagellate forms, and in the lungs small flagellates and oval, non-flagellate dividing parasites occurred. The kidneys did not appear to be infected.

IV. *Some Remarks on the Foregoing Researches.*

A number of interesting points follow from the experiments just recorded and may now be considered, apart from others of economic significance which are dealt with separately in the next section.

The infections obtained in the adult experimental animals were not heavy, thereby differing from those recorded in young animals in our previous paper. Also, in adult hosts the number of observed flagellate forms was relatively few, indeed, much fewer than in the case of young hosts. On the other hand, adult hosts contained more leishmaniform parasites. Doubtless the parasites are more virulent in young hosts, as is the case with Mediterranean Kala-azar in children.

The term "incubation period" has not been used in detailing the experiments because of the difficulty of its exact definition. During the first day after inoculation or feeding with insect flagellates, various stages of them may be observed in the peripheral blood of the adult vertebrate host. These forms are usually rounded or non-flagellate, and differ from flagellate stages which may have been directly introduced. Further, the non-flagellate forms may be seen occasionally in process of division during the first day. After about 18 hours the organisms often disappear, and no parasites may be seen in the blood for a few days. These observations are in complete agreement with those published in 1911 by one of us (H.B.F.) on *Trypanosoma gambiense* and *T. rhodesiense*. Hence, during the so-called incubation period, a few rounded forms of these flagellates (*Herpetomonas, Crithidia, Trypanosoma*) may be found in the peripheral blood. The "incubation period" is often short in young hosts, as was shown in our former experiments.

The appearance and relative position of the blepharoplast in leishmaniform elements vary. It may be either bacilliform or dot-like in shape, and thus resembles that of *Leishmania tropica* more than that often seen in *L. donovani*, though variations occur in all these parasites. *L. tropica* thus seems to have retained more of the variation seen in the original insect flagellate stock from which it was derived.

The ability of species of *Herpetomonas* and *Crithidia* to live in

cold-blooded vertebrates is comparable with that of certain trypanosomes, such as *T. brucei* and *T. lewisi*, which are inoculable into such animals as snakes, lizards and frogs.

It may be added that we have consulted the work of Rocha-Lima (1912) on Blastomycetes. We find that the parasites seen by us are not so numerous as those described by him, and differ in morphology and in bio-chemical reactions, so that they could not be mistaken for yeast-like organisms.

V. *Significance of Experimental Results.*

This subject has already been briefly discussed in our former paper (p. 44), when the zoological importance of the herpetomonad cultural stage was indicated, the finding of a herpetomonad in man by Franchini was noted, and we suggested that canine Kala-azar is really a herpetomoniasis (or leptomoniasis) due to *H. ctenocephali*.

Other points of interest may be considered. From the recent publications of a few observers it is now clear that herpetomonad stages of *Leishmania* may be found in man, at any rate in infections with *L. tropica*. The recent observations of Sergent, Lemaire and Senevet (1914) have demonstrated the presence of a herpetomonad flagellate in cultures of the blood and organs of geckos obtained from areas in Algeria in which oriental sore, due to *L. tropica*, is prevalent. *Phlebotomus* flies, which may harbour a natural herpetomonad, feed on the geckos and on men. Hence animals like geckos may act as reservoirs of leishmaniasis, though Laveran has just recorded his lack of success in attempts to infect geckos and green lizards experimentally with *Leishmania tropica*. Lindsay (1914) writes that the parasite of dermo-mucosal leishmaniasis in Paraguay is believed by native sufferers to be conserved in rattlesnakes, and spread by ticks or flies (*Simulium*) feeding on the reptiles and transferring the parasite to man.

Our experimental results show that there is ground for these hypotheses. We find that insect flagellates belonging to the genera *Herpetomonas* and *Crithidia* have produced infections not only in mammalian hosts, like mice, but also in cold-blooded vertebrates belonging to the Pisces, Amphibia and Reptilia. Furthermore, the flagellates are capable of assuming resting, encysted, postflagellate stages in these hosts.

Regarding the possibility of mice acting as reservoirs of herpetomoniasis (leishmaniasis) in nature, we would draw attention to an observation by Dutton and Todd published in 1903. This observation has apparently been overlooked, though Todd has recently changed his view on the matter. These investigators,

when in Senegambia, found a *Herpetomonas* in the blood of house mice. From the description given, there can be no doubt that the organisms actually were herpetomonads. We have also found herpetomonads occurring naturally in mice in England, and are publishing a paper on the same in *Parasitology*. (See list of References.)

As we have previously stated, we believe that leishmaniases are insect-, or rather, arthropod-borne herpetomoniases. It is highly probable that the maladies have originated from the introduction of flagellates of invertebrates—especially herpetomonads of insects—into vertebrate hosts, in which they have been able to establish and propagate themselves.

Preventive measures in areas where leishmaniases are endemic should be directed against invertebrates harbouring herpetomonad flagellates, especially such invertebrates as may come in contact with men and dogs, and even with domestic vermin like rats and mice. Arthropods need special attention in this connection. It is likely that certain vertebrates, such as reptiles and amphibia—especially those that are insectivorous—may serve as reservoirs of leishmaniases (herpetomoniases). From such reservoirs the herpetomonads may reach man by the agency of ectoparasites or flies, especially such as are sanguivorous.

VI. *Summary.*

1. Herpetomoniasis can be induced in various warm- and cold-blooded vertebrates when the latter are inoculated or fed with herpetomonads occurring in the digestive tracts of various insects. The infection produced and the protozoal parasites found in the vertebrates resemble those of human and canine leishmaniases.

2. An infection can also be induced in certain vertebrates when they are fed or inoculated with *Crithidia gerridis*, and both flagellate and non-flagellate stages occur therein, but no transition to a trypanosome was found.

3. The following Flagellata have proved pathogenic to warm-blooded mammals when the latter have been fed, or inoculated subcutaneously or intraperitoneally with them—*Herpetomonas jaculum*, *H. stratiomyiae*, *H. pediculi* and *Crithidia gerridis*. The hosts used were mice of various ages. That *H. ctenocephali* can infect dogs has already been shown by us.

4. *Herpetomonas jaculum* and *Crithidia gerridis* have also been successfully fed or inoculated into cold-blooded hosts, namely, fishes (*Gasterosteus aculeatus*), frogs, toads, lizards (*Lacerta vivipara*) and grass-snakes (*Tropidonotus natrix*).

5. As we have previously stated, we believe that leishmaniases

are arthropod-borne herpetomoniases, and that these maladies have been evolved from flagellates of invertebrates (especially herpetomonads of insects), which have been able to adapt themselves to life in vertebrates.

6. In areas where leishmaniases are endemic, an examination should be made of all insects and other invertebrates likely to come into contact with men or dogs or rats and mice, in order to ascertain if these invertebrates harbour herpetomonads. Preventive measures should be directed against such invertebrates, especially arthropods. Further, it is likely that certain vertebrates, such as reptiles and amphibia (especially those that are insectivorous), may serve as reservoirs for leishmaniases or, as they should preferably be termed, herpetomoniases. From such reservoirs the herpetomonads may reach man by the agency of ectoparasites or flies, especially such as are sanguivorous.

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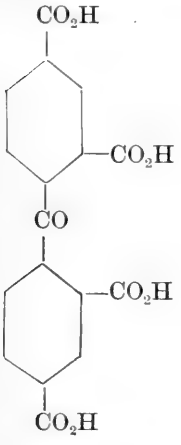
The Ketodilactone of Benzophenone-2-4-2′-4′-tetracarboxylic Acid. (Preliminary Note.) By W. H. Mills, M.A., Jesus College.

[*Read* 10 May 1915.]

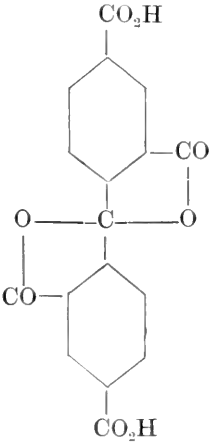
With the object of obtaining a dicyclic compound of the type

which should be capable of resolution into optically active components, the oxidation of di-*m*-xylyl ketone has been investigated. By heating this ketone with dilute (9 per cent.) nitric acid for about 70 hours it was completely converted into a mixture of acids in which dimethylbenzophenone dicarboxylic acids predominated.

On further oxidation with alkaline potassium permanganate the mixed acids gave an excellent yield of benzophenone-2-4-2′-4′-tetracarboxylic acid (I), an easily soluble compound which is transformed with the greatest readiness into its sparingly soluble ketodilactone (II). The transformation is best effected by acidifying the aqueous solution strongly with hydrochloric acid and heating on the water bath. The ketodilactone is then deposited in the course of a few minutes as a heavy crystalline powder (found C = 59·8; H = 2·5; $C_{17}H_8O_8$ requires C = 60·0; H = 2·4 per cent.). This compound melts at 410° and is very sparingly soluble in the common solvents.



I.



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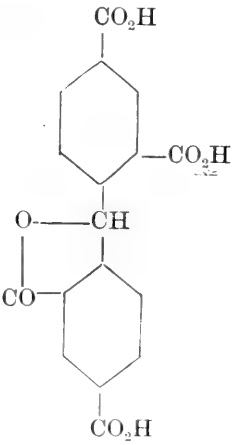

III.

It was converted by heating with phosphoryl chloride and two molecular proportions of phosphorus pentachloride into the acid chloride $C_{15}H_6O_4(COCl)_2$ (found Cl = 18·86; $C_{17}H_6O_6Cl_2$ requires Cl = 18·83 per cent.), from which the esters of the ketodilactone acid can be prepared. The diethyl ester (M.P. 212°) and the dibornyl ester (M.P. 245°) have been prepared and analysed.

The acid is rapidly reduced by zinc dust and ammonia to benzhydrol-2-4-2′-4′-tetracarboxylic acid, which on liberation from its salts very rapidly passes into the lactone III. This compound (found C = 59·4; H = 3·1; $C_{17}H_{10}O_8$ requires C = 59·6; H = 2·9 per cent.) is considerably more soluble in acetic acid or alcohol than the ketodilactone and melts at 311°. Experiments on the resolution of these compounds have been commenced.

In the preparation and investigation of the above-mentioned acid chloride and bornyl ester of the ketodilactone I have had the valuable assistance of Mr A. F. R. Evans, B.A., of Pembroke College, to whom my best thanks are due.

PROCEEDINGS AT THE MEETINGS HELD DURING THE SESSION 1914—1915.

ANNUAL GENERAL MEETING.

October 26th, 1914.

In the Comparative Anatomy Lecture Room.

Dr Shipley, President, in the Chair.

The following were elected Officers for the ensuing year:

President:

Prof. Newall.

Vice-Presidents:

Dr Barnes.
Prof. Seward.
Dr Shipley.

Treasurer:

Prof. Hobson.

Secretaries:

Mr A. Wood.
Mr F. A. Potts.
Mr G. H. Hardy.

Other Members of Council:

Dr Cobbett.
Mr J. Mercer.
Dr Marshall.
Mr F. J. M. Stratton.
Prof. Woodhead.
Mr C. Forster Cooper.
Mr C. E. Inglis.
Dr Duckworth.
Mr J. A. Crowther.
Mr H. H. Brindley.
Dr Fenton.
Mr H. Hamshaw Thomas.

The following was elected a Fellow of the Society:

F. W. Aston, B.A., Trinity College.

The following Communications were made:

1. The Conductivity of Extremely Dilute Acid and Alkali Solutions. By H. H. Paine, M.A., Trinity College, and G. T. R. Evans.

2. Studies in Synthetic Logic. By N. Wiener. (Communicated by Mr G. H. Hardy.)

November 9th, 1914.

In the Cavendish Laboratory.

Professor Newall, President, in the Chair.

The following was elected a Fellow of the Society:

A. M. Smith, M.A., Emmanuel College.

The following was elected an Associate of the Society:

Miss G. L. Buckley, Girton College.

The following Communications were made:

1. Experiments with slow Cathode Rays. By Professor Sir J. J. Thomson.

2. Note on the conditions of stability of electrified drops. By J. Zeleny, B.A. (Communicated by Professor Sir J. J. Thomson.)

November 23rd, 1914.

In the Comparative Anatomy Lecture Room.

Professor Newall, President, in the Chair.

The following Communications were made:

1. Some notes on Insect Parasites. By W. R. Thompson. (Communicated by Mr F. A. Potts.)

2. Some Insect Flagellates introduced into Vertebrates. By Dr H. B. Fantham and Miss Annie Porter.

3. The colour variations of the Fauna associated with Crinoids. (Preliminary note.) By F. A. Potts, M.A., Trinity Hall.

4. The shortest line dividing an area in a given ratio. By N. Wiener. (Communicated by Mr G. H. Hardy.)

5. Preliminary notes on some problems connected with Respiration in Insects generally and true aquatic forms in particular. By G. L. Purser. (Communicated by Mr F. A. Potts.)

February 8th, 1915.

In the Cavendish Laboratory.

Dr Barnes, Vice-President, in the Chair.

The following Communications were made:

1. Theory of the Mobility of Negative Ions. By Professor Sir J. J. Thomson.

2. (1) The determination of the focal length of a thick mirror.

   (2) Experiment on the focal lines formed by refraction at a plane surface.

   (3) Calculation of the electrical resistance of a certain network of conductors.

By Dr G. F. C. Searle.

3. Determination of the thickness of thin plates by an interference method. By C. T. R. Wilson, M.A., Sidney Sussex College.

4. The Wolf-note in bowed stringed Instruments. By G. W. White. (Communicated by Professor Sir J. J. Thomson.)

February 22nd, 1915.

In the Botany School.

Professor Newall, President, in the Chair.

The following Communications were made:

1. On some Fossil Plants from the Devonian rocks of North Devon. By Dr Arber and R. H. Goode, B.A.

2. On some new and rare Jurassic plants from Yorkshire—The male flower of *Williamsonia gigas* (Lind. & Hutt). By H. Hamshaw Thomas, M.A., Downing College.

3. Nomenclature of *Pteris aquilina*. By Dr C. E. Moss.

May 10th, 1915.

In the University Chemical Laboratory.

Professor Newall, President, in the Chair.

The following Communications were made:

1. (1) The Ketodilactone of Benzophenone-2-4-2′-4′-tetracarboxylic Acid.

   (2) The Synthesis of 1-5-Dibromo-3-isopropylpentane.

By W. H. Mills, M.A., Jesus College.

2. Further experimental researches on Insect Flagellates introduced into Vertebrates. By Dr H. B. Fantham and Miss Annie Porter.

3. Note on Dr Searle's experiment on the harmonic motion of a rigid body. By Sir G. Greenhill.

4. On the Electrification given to the Air by a Steam Jet. By W. A. D. Rudge, M.A., St John's College.

PROCEEDINGS

OF THE

Cambridge Philosophical Society.

Experiments with a prism of small angle. By G. F. C. Searle, Sc.D., F.R.S., University Lecturer in Experimental Physics, Fellow of Peterhouse.

[*Read* 23 February 1914.]

§ 1. *Introduction.* The prisms of small angles supplied by opticians for use in spectacles are convenient in a number of optical experiments. When the refracting angle of a prism is so small that the circular measure, the tangent and the sine of the angle may be treated as identical to the accuracy which can be reached in the measurements, the mathematical calculations become so simple that the attention of the student is not diverted by them from the physical phenomena of refraction and deviation which he is studying. There is a close connexion between a prism of small angle and a simple lens of small aperture, for, when spherical aberration is neglected, the effect of the lens on rays proceeding from a point on its axis may be found by treating the part of the lens traversed by one of these rays as if it were part of a prism of small angle. Many lens problems are best solved by considering the deviation of a ray which meets a thin lens, or the principal planes of a thick lens, at a definite distance from the axis, and thus experiments such as those described below form a useful introduction to work with lenses.

The apparatus is simple and can be freely handled by the student without risk of injury. Though the goniometer is somewhat inferior in accuracy to a good spectrometer fitted with a pair of verniers, it has the great advantage that the readings can be very quickly taken, with the result that the student spends his time in studying optics rather than verniers.

§ 2. *The auto-collimating goniometer.* The goniometer used in measuring the angles in the experiments to be described is shown in Fig. 1; it was designed in conjunction with W. G. Pye and Co. The base is formed of a strip of mahogany furnished at one end with a spherical pivot, and at the other with a cross-bar carrying a scale. Angles are measured by means of a moveable arm which turns at one end about the pivot, while the other end moves over the scale on the cross-bar. The optical system consists of an

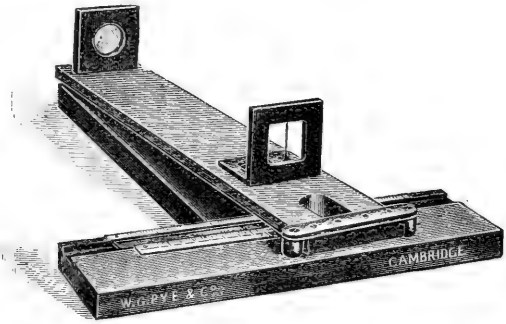

Fig. 1.

achromatic lens, about 35 cm. in focal length, fixed to the arm above the pivot and of a fine vertical wire held in an adjustable frame attached to the other end of the arm; the frame is adjusted so that the wire is in the focal plane of the lens.

The spherical pivot is a phosphor-bronze ball attached to an adjustable fitting. The ball enters a conical hole turned out of a block of brass attached to the arm; the other end of the arm carries two brass feet which rest upon the cross-bar.

The scale is provided with an anti-parallax mirror and is divided into millimetres and the ball is adjusted so that its centre is 40 cm. from the edge of the scale. The readings are taken by means of a fine wire passing across an opening and stretched by a spring. As the scale can be read to $\frac{1}{100}$ cm., the angle can be read to $\frac{1}{4000}$ radian or to about $\frac{1}{70}$ degree.

The goniometer is converted into an auto-collimating instrument by the addition of the fitting shown in Fig. 2. The bar EF is clamped to the frame holding the vertical wire of the goniometer by the screws G, H. A vertical slit is cut in the upper edge of the bar and this slit is covered by a small totally reflecting prism T. When light from a source S falls upon the prism, it is internally reflected and passes out through the slit past the vertical wire WW. The beam of light then passes through the lens of the goniometer; if it suffers reflexion at a suitably placed

plane mirror, those rays which fall normally upon the mirror will retrace their paths, and, after passing a second time through the lens, will come to a focus at the point in the focal plane from which they started. Other points in the focal plane will have

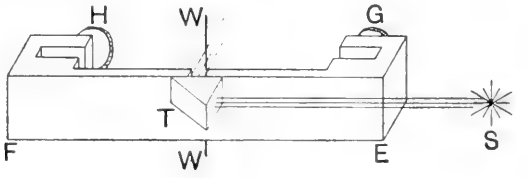

Fig. 2.

inverted images in that plane and thus an inverted image of the slit and wire will be seen. By properly adjusting the mirror about a horizontal axis, the lower edge of the image of the slit may be made to lie on the upper edge of the bar *EF* (Fig. 2), and then, by adjusting the arm of the goniometer or by turning the mirror about a vertical axis, the image of the wire may be made to coincide with the wire itself.

The goniometer may, conveniently, be bolted to a table carried by a Compound Laboratory Stand (W. G. Pye and Co.); its height above the table is then adjustable.

The goniometer does not measure angles directly but their tangents. If the indicating wire crosses the edge of the scale at a distance x cm. from the central division and if the angle between the displaced and the central positions of the arm is θ radians, then $\tan\theta = x/40$, if the distance from the pivot to the edge of the scale is 40 cm. The scale extends 10 cm. on either side of its central division so that the greatest value of $\tan\theta$ is 0·25. When $\tan\theta$ is known, θ can be found in degrees by trigonometrical tables or in radians by the series

$$\theta = \tan\theta - \tfrac{1}{3}\tan^3\theta + \tfrac{1}{5}\tan^5\theta - \ldots, \quad \ldots\ldots\ldots\ldots(1)$$

provided $|\tan\theta| < 1$.

The following Table, calculated to five decimal places by aid of the series, enables θ to be easily found when $\tan\theta$ is known. It gives the value of c, the quantity which must be subtracted from $\tan\theta$ in order to obtain the value of θ in radians. For values of $\tan\theta$ not given in the Table, interpolation may be used. Thus, when $\tan\theta = 0{\cdot}205$, $c = 0{\cdot}00280$ and thus

$$\theta = \tan\theta - c = 0{\cdot}205 - 0{\cdot}00280 = 0{\cdot}20220 \text{ radians.}$$

The Table gives c with greater accuracy than the present experiments demand, but the extra figures are given in the hope that it may be useful for other purposes.

tan θ	c	tan θ	c	tan θ	c
0·01	0·00000	0·10	0·00033	0·19	0·00224
0·02	0·00000	0·11	0·00044	0·20	0·00260
0·03	0·00001	0·12	0·00057	0·21	0·00301
0·04	0·00002	0·13	0·00072	0·22	0·00345
0·05	0·00004	0·14	0·00090	0·23	0·00393
0·06	0·00007	0·15	0·00111	0·24	0·00446
0·07	0·00011	0·16	0·00134	0·25	0·00502
0·08	0·00017	0·17	0·00161		
0·09	0·00024	0·18	0·00191		

§ 3. *Measurement of the angle and the refractive index of a prism by the auto-collimating goniometer.* Let ABC (Fig. 3) be a section of the prism by a principal plane and let the angle

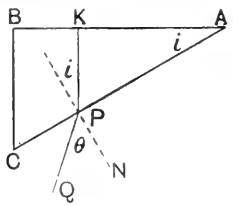

Fig. 3

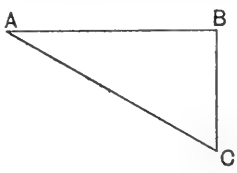

Fig. 4.

BAC of the prism be i radians. Let P be a point on the face AC and let PK be perpendicular to the face AB. Let the ray KP on passing out of the glass into the air be refracted along PQ, making an angle θ with the normal PN. Then, since PK makes an angle i with the normal, we have, if μ be the refractive index of the prism,

$$\sin\theta = \mu \sin i.$$

The two directions PN and PQ are readily identified by optical means, for PN is the direction of a parallel beam of rays which returns along its own path after reflexion at AC, and PQ is the direction of a parallel beam which returns along its own path after two refractions at AC and one reflexion at AB.

The angle i of the prism is easily found. For, if the prism be turned through two right angles about an axis perpendicular to the plane of the face AB, so that the section of the prism comes into the position shown in Fig. 4, the normal to the face AC

will then make an angle $2i$ with its former direction. Hence this motion of the prism will entail a change of direction of $2i$ in the parallel beam of rays which falls normally upon the face AC.

The prism is held in a simple geometrical clamp (Fig. 5). Three bicycle balls are soldered to a brass plate and one face

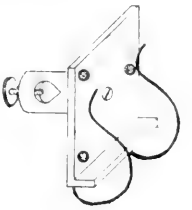

Fig. 5.

of the prism (AB in Fig. 3) is kept in contact with the spheres by two steel springs. This spring clamp is attached to a horizontal rod carried by a heavy stand. The screw at the centre of the brass plate allows the plate to be adjusted so that the ledge at the bottom of the plate is horizontal. If the ends of the prism have been made perpendicular to the refracting edge (which is not always the case), this will secure that the prism is properly placed.

The goniometer is fixed at such a height above the table that light from a sodium flame passes properly through its reflecting prism. The clamp holding the prism to be tested is then set to the corresponding height, and the clamp is placed close to the lens of the goniometer and is turned about the horizontal rod supporting it so that, when the arm of the goniometer is suitably directed, an image of the illuminated slit is seen just above the totally reflecting prism. There are two positions of the arm in which an image is seen. In one position the light is reflected at the front face of the prism, in the second it is reflected at the back face; the first position corresponds to PN in Fig. 3 and the second to PQ. A reference to Fig. 3 will enable the observer to distinguish between the two images. [The image formed by reflexion at the back face of the prism can be destroyed by smearing vaseline over that face.]

It may be found that there is slight parallax between the image of the wire and the wire itself when the prism is used. If the setting of the wire is perfect as judged by a good plane mirror, this indicates that one or both faces of the prism is not quite plane.

If the two images are not at the same height above the bar carrying the totally reflecting prism, the prism to be tested is not quite accurately adjusted. The error may be corrected by turning

the prism through a small angle about a horizontal axis perpendicular to the plane of the three spheres.

The marked face of the prism is placed against the spheres and the two goniometer readings corresponding to reflexion at the front face and to reflexion at the back face are taken. The angle between the two positions of the arm is the angle θ in Fig. 3. Keeping the marked face still in contact with the spheres, the prism is turned through 180° about a horizontal axis perpendicular to the plane of the spheres and the observations are repeated, thus furnishing a second value of θ; care must, of course, be taken not to move the clamp. The angle between the two positions of the arm corresponding to reflexion at the front face is equal to $2i$. The goniometer readings are reduced to radians by aid of the Table where necessary. The observations may be repeated with the unmarked face of the prism in contact with the spheres; to ensure quite independent readings, the goniometer stand may be slightly moved from its first position.

The refractive index is then found by the formula

$$\mu = \sin\theta/\sin i. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

§ 4. *Practical example.* The following observations were made by G. F. C. Searle.

Distance of goniometer scale from pivot = 40·00 cm.

The central reading of the goniometer is 10·00 cm.

The letters F and B indicate readings corresponding to reflexion at the front and back faces of the prism respectively.

(i) Marked face in contact with spheres:

Mark to right.		Mark to left.	
$F_1 = 7{\cdot}30$ cm.	$B_1 = 11{\cdot}59$ cm.	$F_2 = 12{\cdot}92$ cm.	$B_2 = 8{\cdot}62$ cm.

(ii) Unmarked face in contact with spheres:

Mark to right.		Mark to left.	
$F_3 = 7{\cdot}08$ cm.	$B_3 = 11{\cdot}38$ cm.	$F_4 = 12{\cdot}70$ cm.	$B_4 = 8{\cdot}41$ cm.

The deviation of the arm from its central position for F_1 is $-2{\cdot}70$ cm. and the angle is $-\tan^{-1}(2{\cdot}70/40)$ or $-\tan^{-1}0{\cdot}06750$. By the Table the angle is $-(0{\cdot}06750 - 0{\cdot}00010)$ or $-0{\cdot}06740$ radians. In the same way F_2 corresponds to a deviation of 0·07287 radians, and hence

$$2i = 0{\cdot}06740 + 0{\cdot}07287 = 0{\cdot}14027 \text{ radians.}$$

Similarly we find from F_1 and B_1,

$$\theta = 0{\cdot}10713,$$

and from F_2 and B_2, $\theta = 0{\cdot}10736,$

the mean being 0·10725 radians.

The readings F_3, B_3 and F_4, B_4 give the values

$$2i = 0{\cdot}14027, \quad \theta = 0{\cdot}10725 \text{ radians.}$$

The mean values are

$$i = 0{\cdot}07014 \text{ radians} = 4^\circ\ 1'{\cdot}1, \quad \theta = 0{\cdot}10725 \text{ radians} = 6^\circ\ 8'{\cdot}7.$$

Hence, by (2),

$$\mu = \sin\theta/\sin i = 1{\cdot}528.$$

§ 5. *Determination of the angle and refractive index of a prism by aid of a liquid of known index.* In the last experiment no assumption was made as to the magnitude of the angle of the prism. In the following experiment, however, in order to simplify the mathematics, it is necessary that the angles concerned should be "small."

We shall first calculate the deviation when a ray passes with small angles of incidence and refraction from air into a medium of index μ. Let PQR (Fig. 6) be a ray making angles θ and ϕ

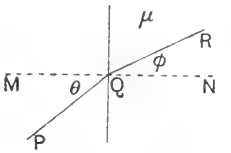

Fig. 6.

with the normal MQN in the air and in the medium. Then $\sin\theta = \mu \sin\phi$. But

$$\theta = \sin\theta + \frac{1}{2}\cdot\frac{1}{3}\sin^3\theta + \frac{1.3}{2.4}\cdot\frac{1}{5}\sin^5\theta + \ldots, \quad \ldots\ldots(3)$$

and $\sin\phi = \phi - \frac{1}{6}\phi^3$, as far as ϕ^3. Hence, to this order,

$$\theta = \mu\sin\phi + \tfrac{1}{6}(\mu\sin\phi)^3 = \mu(\phi - \tfrac{1}{6}\phi^3) + \tfrac{1}{6}\mu^3\phi^3$$
$$= \mu\phi + \tfrac{1}{6}\mu(\mu^2 - 1)\phi^3. \quad \ldots\ldots(4)$$

If the deviation be D, we have $D = \theta - \phi$, and thus

$$D = (\mu - 1)\phi + \tfrac{1}{6}\mu(\mu^2 - 1)\phi^3. \quad \ldots\ldots\ldots\ldots\ldots(5)$$

Next consider the passage of a ray in a principal plane through a prism of index μ and of small angle i. If the ray passes through the prism symmetrically, the angles which it makes in the medium of index μ with the two normals are each $\frac{1}{2}i$. If the path $PQRS$ (Fig. 7) is unsymmetrical, these angles will be $\frac{1}{2}i + \eta$ and $\frac{1}{2}i - \eta$,

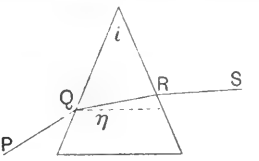

Fig. 7.

where η is the angle between the unsymmetrical path and a symmetrical path in the medium. For the refraction at Q, where the value of ϕ is $\frac{1}{2}i + \eta$, we have, by (5),

$$D_1 = (\mu - 1)(\tfrac{1}{2}i + \eta) + \tfrac{1}{6}\mu(\mu^2 - 1)(\tfrac{1}{2}i + \eta)^3, \quad \ldots\ldots(6)$$

and for the refraction at R

$$D_2 = (\mu - 1)(\tfrac{1}{2}i - \eta) + \tfrac{1}{6}\mu(\mu^2 - 1)(\tfrac{1}{2}i - \eta)^3. \quad \ldots\ldots(7)$$

The whole deviation, D, is the sum of D_1 and D_2 and hence

$$\begin{aligned} D &= (\mu - 1)\, i + \tfrac{1}{24}\mu(\mu^2 - 1)\, i\,(i^2 + 12\eta^2) \\ &= (\mu - 1)\, i\,\{1 + \tfrac{1}{24}\mu(\mu + 1)(i^2 + 12\eta^2)\}. \quad \ldots\ldots\ldots(8) \end{aligned}$$

The term involving η^2 shows how lack of symmetry increases the deviation. When both i and η are small, we may write*

$$D = (\mu - 1)\, i, \quad \ldots\ldots\ldots(9)$$

which is the formula we shall use in the experiment.

The prism is placed with its edge vertical in a tank (Fig. 8) with parallel glass sides containing a liquid of refractive index μ_1. If the sides of the tank are plates of plane-parallel glass, they will not affect the deviation, and we may treat the system as if it were a glass prism in a block of water. We then have three prisms, viz. a glass prism of angle i and two liquid prisms whose angles are β and γ (Fig. 8), the sum of β and γ being i, β and γ being counted positive when the refracting angles of these prisms point in the opposite direction to that of the glass prism. If a ray now

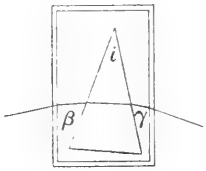

Fig. 8.

passes through the system, falling nearly normally upon the surface of the liquid, it will pass nearly symmetrically through each of the prisms and the formula (9) may be applied to each prism. If D_1 be the resultant deviation, and if D_1 be counted positive when it is in the same direction as that due to the glass prism alone, we have, since $\beta + \gamma = i$,

$$D_1 = (\mu - 1)\, i - (\mu_1 - 1)\,\beta - (\mu_1 - 1)\,\gamma = (\mu - 1)\, i - (\mu_1 - 1)\, i.$$

Hence

$$D_1 = (\mu - \mu_1)\, i. \quad \ldots\ldots\ldots(10)$$

Equations (9) and (10) enable us to find i and μ, if we know μ_1 and observe D and D_1. Since by (9) and (10)

$$\frac{\mu - 1}{\mu - \mu_1} = \frac{D}{D_1}, \quad \ldots\ldots\ldots(11)$$

* When a ray passes symmetrically through a prism, the deviation is given by $\sin \tfrac{1}{2}(D + i) = \mu \sin \tfrac{1}{2}i$. When i is small, we can write $D + i = \mu i$ or $D = (\mu - 1)\, i$, but this result does not show how the want of symmetry affects the deviation.

we find $$\mu = \frac{\mu_1 D - D_1}{D - D_1}. \quad \text{.....} \quad \text{.....} (12)$$

Subtracting (10) from (9), we find $D - D_1 = (\mu_1 - 1)\, i$, and thus

$$i = \frac{D - D_1}{\mu_1 - 1}. \quad \text{.....} \quad \text{.....} (13)$$

§ 6. *Experimental details.* The measurements are made by the goniometer described in § 2. The collimator AB (Fig. 9) has a pair of cross wires at A. These wires are fixed for their protection to the inner end of a short tube which slides in the main

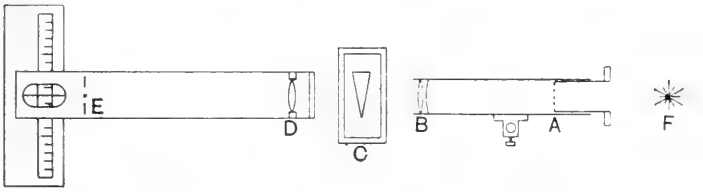

Fig. 9.

tube of the collimator, and the wires are adjusted to lie in the focal plane of the achromatic lens B (of 15 to 20 cm. focus). The tank C has sides of plane glass, but it is not necessary that the two sides should be accurately parallel. The collimator is illuminated by a sodium flame F. If the goniometer DE is properly placed, an image of the vertical wire at A will coincide with the vertical wire of the goniometer at E. The tank contains the liquid of refractive index μ_1.

The prism is first placed between the collimator and the goniometer, the tank being removed, and a reading of the goniometer is taken. The prism is then turned through 180° about a vertical axis and a second reading is taken. The difference between the two readings corresponds to $2D$.

The tank is then put into place and is filled with water, and the prism is placed in the tank, as indicated in Fig. 9. The goniometer reading is taken and then, without disturbing the tank, the prism is turned through 180° and a second reading is taken, the difference corresponding to $2D_1$. This reversal of the prism eliminates any error due to want of parallelism between the sides of the tank.

If x and x_1 be the changes of goniometer reading corresponding to D and to D_1, the angles will be so small with a suitable prism that the corrections given by the Table in § 2 will be negligible, and thus we may write

$$D = x/2l, \qquad D_1 = x_1/2l,$$

where l is the length from the centre of the pivot to the edge of the goniometer scale, and the angles are expressed in radians. Equations (12) and (13) then give μ and i. For water

$$\mu_1 = 1{\cdot}3334 - 0{\cdot}00009\,(t - 15),$$

where t is the centigrade temperature. For temperatures near 15° C. it will suffice to take $\mu_1 = \frac{4}{3}$ and $\mu - 1 = \frac{1}{3}$.

The method may be employed to find the refractive index of a liquid such as a salt solution. If this has the refractive index μ_2, and if $2D_2$ be the change of deviation when the prism is turned round in the solution, which is contained in the tank, then, by (13),

$$D - D_2 = (\mu_2 - 1)\,i,$$

and thus

$$\mu_2 = 1 + (D - D_2)/i. \quad \ldots\ldots\ldots\ldots\ldots\ldots(14)$$

If the auto-collimating device of the goniometer is used, the collimator may be dispensed with. A plane mirror of good quality is placed with its face approximately perpendicular to the axis of the goniometer arm when the latter is in its central position. The goniometer wire is then made to coincide with its own image in each case.

§ 7. *Practical example.* The following observations were made, using the same prism as in § 4. The auto-collimating method was used. The value of l was 40 cm.

Observations for D. Prism in air.
Edge towards left. Readings 11·37, 11·37, 11·37. Mean 11·370 cm.
Edge towards right. Readings 8·41, 8·40, 8·40. Mean 8·403 cm.
Hence $D = \frac{1}{2}\,(11{\cdot}370 - 8{\cdot}403)/40 = 2{\cdot}967/80 = 0{\cdot}03709$ radians.
Observations for D_1. Prism in water at 20° C; $\mu_1 = \frac{4}{3}$.
Edge towards left. Readings 10·39, 10·38, 10·38. Mean 10·383 cm.
Edge towards right. Readings 9·30, 9·30, 9·30. Mean 9·300 cm.
Hence $D_1 = \frac{1}{2}\,(10{\cdot}383 - 9{\cdot}300)/40 = 1{\cdot}083/80 = 0{\cdot}01354$ radians.
By (13), we have for the angle of the prism

$$i = \frac{D - D_1}{\mu_1 - 1} = 3\,(0{\cdot}03709 - 0{\cdot}01354) = 0{\cdot}07065 \text{ radians} = 4^\circ\ 2'{\cdot}9.$$

By (12), we have for the refractive index of the prism

$$\mu = \frac{\mu_1 D - D_1}{D - D_1} = \frac{4 \times 0{\cdot}03709/3 - 0{\cdot}01354}{0{\cdot}03709 - 0{\cdot}01354} = 1{\cdot}525.$$

Observations for D_2. Prism in saturated solution of sodium chloride.
Edge towards left. Readings 10·28, 10·28, 10·27. Mean 10·277 cm.
Edge towards right. Readings 9·44, 9·45, 9·45. Mean 9·447 cm.
Hence $D_2 = \frac{1}{2}\,(10{\cdot}277 - 9{\cdot}447)/40 = 0{\cdot}830/80 = 0{\cdot}01038$ radians.
By (14), we have for the refractive index of the salt solution

$$\mu_2 = 1 + (D - D_2)/i = 1 + 0{\cdot}02671/0{\cdot}07065 = 1{\cdot}378.$$

§ 8. *Determination of the angle and refractive index of a prism by primary and secondary images.* When a ray which enters a prism ABC by the face AB meets the face AC, the greater part of the light is refracted out through that face, but some is reflected back into the prism. If this reflected light

strikes the face AB, some of it is reflected at that face and again falls on AC, where the greater part of it suffers refraction and passes out of the prism. If the angle of the prism is small enough, some of the light may suffer 4, 6, ... internal reflexions, with corresponding emergent rays, but the light is so weakened at each reflexion that the images due to rays which have suffered more than two reflexions will not be easily seen unless the source of light is very powerful.

The minimum deviation of a ray which suffers two reflexions is easily found. Let $PQRSTU$ (Fig. 10) be the ray. Since the path is symmetrical $AQ = AT$ and $AS = AR$, so that SR makes an angle $\frac{1}{2}i$ with the normal to AC at R. But the normal at R bisects SRQ and hence $QRT = \frac{1}{2}\pi - \frac{1}{2}i$. Thus

$$AQR = QRT - QAR = \tfrac{1}{2}\pi - \tfrac{3}{2}i,$$

and hence the angle between RQ and the normal NQ at Q is $\frac{3}{2}i$.

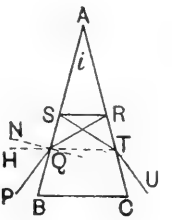

Fig. 10.

If TQH be a straight line, then $PQH = \frac{1}{2}D'$, where D' is the deviation. Then the angle between PQ and the normal NQ is $\frac{1}{2}D' + \frac{1}{2}i$. Hence, by Snell's law,

$$\sin \tfrac{1}{2}(D' + i) = \mu \sin \tfrac{3}{2}i. \quad \ldots\ldots\ldots\ldots\ldots\ldots(15)$$

When i is so small that we can use the angles instead of their sines, we obtain

$$D' = (3\mu - 1)\, i. \quad \ldots\ldots\ldots\ldots\ldots\ldots(16)$$

The minimum deviation (D) for a ray which has not suffered reflexion is approximately

$$D = (\mu - 1)\, i. \quad \ldots\ldots\ldots\ldots\ldots\ldots(17)$$

From (16) and (17) we find

$$\mu = \frac{D' - D}{D' - 3D}, \quad \ldots\ldots\ldots\ldots\ldots\ldots(18)$$

and

$$i = \tfrac{1}{2}(D' - 3D). \quad \ldots\ldots\ldots\ldots\ldots\ldots(19)$$

The measurements are carried out just as in § 6, using either a collimator or the auto-collimating device and a plane mirror. With $\mu = \frac{3}{2}$, the deviation D' is seven times as great as D. A bright source of light will be required. Unless the inside of

the collimator tube is coated with a good "optical black," so much light will be reflected from the sides of the tube that it will be difficult to see the faint secondary image. The difficulty may be met by placing the goniometer so that there is a considerable distance (30—50 cm.) between its lens and that of the collimator. The prism is placed as close as convenient to the lens of the goniometer. Under these conditions, when the goniometer arm is directed to view the secondary image, the light reflected from the sides of the tube will give little trouble.

§ 9. *Practical example.* The following observations were made, using the same prism as in § 4. A collimator was used.

Edge of prism turned towards right.

Goniometer reading for direct rays, 4·04 cm.

Reading for primary image, 5·57 cm. Reading for secondary image, 14·16 cm.

Central reading of goniometer scale, 10·00 cm. Length of arm, 40 cm.

The tangents of the three deviations from the central position are

$(10-4{\cdot}04)/40=0{\cdot}1490$; $(10-5{\cdot}57)/40=0{\cdot}1108$; $(14{\cdot}16-10)/40=0{\cdot}1040$.

The corresponding angles are, by the Table in § 2,

$0{\cdot}1490-0{\cdot}0011=0{\cdot}1479$; $0{\cdot}1108-0{\cdot}0004=0{\cdot}1104$; $0{\cdot}1040-0{\cdot}0003=0{\cdot}1037$.

Hence $$D=0{\cdot}1479-0{\cdot}1104=0{\cdot}0375 \text{ radians},$$
$$D'=0{\cdot}1479+0{\cdot}1037=0{\cdot}2516 \text{ radians}.$$

When the edge of the prism was turned to the left, similar observations gave the values $D=0{\cdot}0372$, $D'=0{\cdot}2517$ radians.

The mean values are $D=0{\cdot}03735$, $D'=0{\cdot}25165$ radians.

Hence, by (18) and (19),

$$\mu=\frac{D'-D}{D'-3D}=\frac{0{\cdot}2143}{0{\cdot}1396}=1{\cdot}535,$$
$$i=\tfrac{1}{2}(D'-3D)=\tfrac{1}{2}\times 0{\cdot}1396=0{\cdot}0698 \text{ radians}$$
$$=4^\circ\ 0'{\cdot}0.$$

§ 10. *Determination of the angle between two nearly perpendicular mirrors.* If two plane mirrors are inclined to each other at exactly a right angle, any ray which suffers two reflexions, one at each of the mirrors, leaves the second mirror parallel to its original direction. When the angle between the mirrors is not exactly 90°, the initial and final rays are no longer parallel. If the initial ray is parallel to a principal plane of the mirrors, *i.e.* to a plane which cuts both mirrors at right angles, the angle between the two rays is twice the difference between 90° and the angle between the mirrors. Let AB, AC (Fig. 11) be a section of the mirrors by a principal plane and let the angle BAC be $\frac{1}{2}\pi+\theta$, so that it exceeds a right angle. Let the ray be $PQRS$ and let $BQP=\alpha$. Then, if PQ and SR meet in F, $FQR=2\alpha$. Further,

$$ARQ=\pi-AQR-QAR=\pi-\alpha-(\tfrac{1}{2}\pi+\theta)=\tfrac{1}{2}\pi-\alpha-\theta,$$

and hence $$FRQ=2ARQ=\pi-2\alpha-2\theta.$$

Thus $PFS = \pi - FQR - FRQ = 2\theta$. It is important to notice that the angle between PQ and RS does not depend upon the angle of incidence of the initial ray at Q.

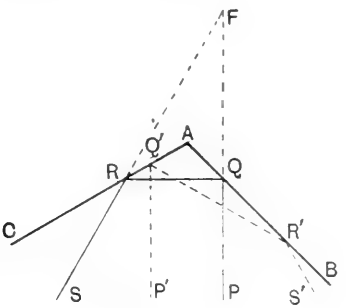

Fig. 11.

In Fig. 11 the ray PQ falls first on the mirror AB. If, however, a ray $P'Q'$ parallel to PQ falls first on AC, it will give rise to a ray $R'S'$ which will be inclined to the ray $P'Q'$ at the angle 2θ, but the deviation of $R'S'$ from $P'Q'$ will be in the opposite direction to the deviation of RS from PQ. Hence a *beam* of rays parallel to PQ, which is wide enough to fall upon *both* mirrors, gives rise to *two* beams parallel to RS and $R'S'$, these beams deviating by equal angles but in opposite directions from PQ. In this case we could find by how much the angle BAC differs from 90°, but we could not decide whether BAC is greater or less than 90°. To do this, we must ensure that the incident light falls on only *one* of the two mirrors—say on AB.

Thus we can find the value of θ if we can measure the angle PFS between the initial and final rays. This can be done by the auto-collimating goniometer.

§ 11. *Experimental details.* The apparatus is arranged as indicated diagrammatically in Fig. 12. Here AB, AC are the two mirrors, LM is the lens of the goniometer, T is the totally reflecting prism of the auto-collimating device (see Fig. 2), and S is a source of light, preferably a small incandescent gas burner. When the burner is so close to T that the rays which pass through T fill the lens LM with light, *two* images of the wire W will be seen, the wire being midway between the images. If, however, the burner is placed at some distance from T in such a position that the rays transmitted by T fall only on the mirror AB (as indicated in Fig. 12), only *one* image will be seen. If, as in Fig. 12, this single image U is to the left of W, then the angle BAC is greater than a right angle; if U is to the right of W, the angle is less than a right angle. I owe this suggestion to Mr S. D. Chalmers. When the burner is nearly in the proper position, the

adjustment may be completed by moving the goniometer arm. The observation is facilitated if a slit 3 or 4 mm. in width is placed near the burner.

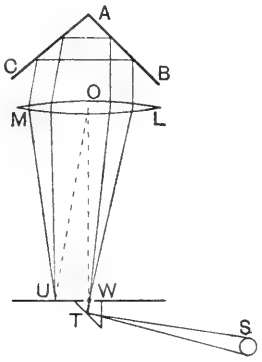

Fig. 12.

For measuring the angle, a piece of glass millimetre scale is held against the frame supporting the vertical wire W of the goniometer, so that W is in contact with the divided face of the scale. To increase the distance to be measured, the flame should be placed so that *both* images are visible. It will be found that the two images move together with W when the goniometer arm is moved.

If O (Fig. 12) be that nodal point of the lens which corresponds to the focal plane WU, then $OW = f$, the focal length. By the property of nodal points, OW and OU are parallel to the two parallel beams on the other side of the lens. Thus OW and OU correspond to PQ and RS in Fig. 11, and hence, in the case of Fig. 12, the angle (ϕ) between the mirrors is $\phi = \frac{1}{2}\pi + \frac{1}{2}UOW$. If $WU = y$, then

$$\phi = \tfrac{1}{2}\pi + y/2f. \quad \text{.....(20)}$$

The focal length of the lens may be measured by any of the usual methods, but it may be conveniently found without disturbing the goniometer. A single plane mirror is substituted for the double mirror of Fig. 12. As the arm of the goniometer is moved through any angle γ, the image of the wire moves across the focal plane, the displacement of the image relative to the wire being the same as if the arm had been at rest and the mirror had been turned through the angle γ. In this case the reflected beam is turned through an angle 2γ. Thus, if a change of goniometer reading of a cm. corresponds to a change of reading of the image on the glass scale of b cm. and if l cm. is the distance from the centre of the pivot to the scale, we have $b/f = 2a/l$ or

$$f = bl/2a. \quad \text{.....(21)}$$

In the goniometers used at the Cavendish Laboratory, l is adjusted to be 40·00 cm.

A mirror system may be constructed of two mirrors fixed to a block of wood. If glass silvered at the back is used there will be a number of reflected images, which will not coincide unless the mirrors are absolutely "plane-parallel." There will be no difficulty, however, in deciding if an image corresponds to two reflexions, one at each of the silvered surfaces, as this image is much brighter than the others. Multiple reflexions may be avoided by using two pieces of unsilvered plate glass, covered at the back with black varnish, or even, as a temporary measure, with vaseline.

§ 12. *Practical example.* A mirror system consisting of two glass plates fixed to a block of wood was used.

Distance of centre of pivot from goniometer scale $= l = 40{\cdot}00$ cm.

The focal length of the goniometer lens was found by the method described in § 11.

Reading of glass scale	Reading of goniometer scale	Reading of glass scale	Reading of goniometer scale	Change of goniometer reading for 1·5 cm. on glass scale
6·5 cm.	9·35 cm.	8·0 cm.	10·21 cm.	0·86 cm.
7·0	9·64	8·5	10·50	0·86
7·5	9·93	9·0	10·78	0·85
8·0	10·21	9·5	11·06	0·85

Mean 0·855 cm.

Hence, by (21), $f = bl/2a = 1{\cdot}5 \times 40/1{\cdot}710 = 35{\cdot}09$ cm.

The readings on the glass scale of the images due to the double mirror were 7·98 and 7·65 cm.; half the difference is 0·165 cm. When the burner was moved away from the mirror system, as in Fig. 12, so as to illuminate the mirror AB only, the surviving image U was to the right of W, *i.e.* on the same side as the burner. The angle between the mirrors is therefore *less* than a right angle. Hence, by (20), if ϕ be the angle between the mirrors,

$$\phi = \tfrac{1}{2}\pi - y/2f = \tfrac{1}{2}\pi - 0{\cdot}165/70{\cdot}18 = \tfrac{1}{2}\pi - 0{\cdot}002351$$
$$= 90^\circ - 8'{\cdot}1 = 89^\circ\ 51'{\cdot}9.$$

§ 13. *Determination of the angle of a prism of nearly* 90°. The method of §§ 10, 11 is easily adapted to the measurement of such a prism, provided the face opposite the reputed right angle is polished, as it must be if the prism is intended to change the direction of rays by refraction. Let ABC (Fig. 13) be a principal section of the prism, the angle A being nearly $\frac{1}{2}\pi$. Let $PQRSXY$ be the path of a ray, the incidence at Q being nearly normal. By § 10, if $BAC = \frac{1}{2}\pi + \theta$, the angle between QR and XS is 2θ, and, if

θ is positive, the point of intersection of QR and XS is on the same side of RS as A is, and this is true whatever the angle of incidence of QR at R. If BAC is exactly $\frac{1}{2}\pi$, QR and SX are

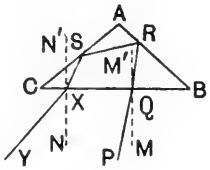

Fig. 13.

parallel and then PQ and XY are parallel also. When, however, QR and SX are not parallel, we must take account of the refractions at Q and X. Let MQM', NXN' be the normals at Q and X. Then $\sin PQM = \mu \sin RQM'$ and $\sin YXN = \mu \sin SXN'$, where μ is the refractive index of the prism. Now it is easy to place the prism so that the ray PQ which falls upon it from the autocollimating goniometer is nearly normal to BC, and then, if BAC is nearly $\frac{1}{2}\pi$, the ray XY will also be nearly normal to BC. We may then use the angles instead of their sines and then

$$PQM = \mu RQM' \text{ and } YXN = \mu SXN',$$

and thus the angle between PQ and YX is μ times the angle between QR and SX, *i.e.* it is $2\mu\theta$. The angle between PQ and YX is measured by the goniometer by the method of §§ 10, 11 in exactly the same way as for the double mirror. If this angle be ψ radians, then $\psi = 2\mu\theta$, and the error in the angle BAC is $\psi/2\mu$ radians. The method indicated in Fig. 12 is used to decide whether BAC is greater or less than a right angle. The refractive index of the prism must be found by an ordinary spectrometer.

§ 14. *Practical example.* The following results were obtained with a prism whose angles were nearly 90°, 45°, 45°. The same goniometer was used as in § 12.

The refractive index of the prism was measured by a spectrometer. Using one of the smaller angles, the angle (i) was found to be 44° 59′ and the minimum deviation (D) was 25° 32′. Hence

$$\mu = \frac{\sin \frac{1}{2}(D+i)}{\sin \frac{1}{2} i} = \frac{\sin 35° 15'{\cdot}5}{\sin 22° 29'{\cdot}5} = 1{\cdot}509.$$

The readings on the glass scale of the images due to the prism were 7·70 and 7·88 cm.; half the difference is 0·09 cm. The value of ψ is

$$0{\cdot}09/f = 0{\cdot}09/35{\cdot}09 = 0{\cdot}00256 \text{ radians.}$$

When the burner was placed so as to illuminate the face AB and not the face AC, the surviving image U (Fig. 12) was to the left of W, *i.e.* on the opposite side to the burner. The angle of the prism is therefore *greater* than a right angle. Hence, if A be the angle BAC,

$$A = \tfrac{1}{2}\pi + \psi/2\mu = \tfrac{1}{2}\pi + 0{\cdot}00256/3{\cdot}018 = \tfrac{1}{2}\pi + 0{\cdot}00085 \text{ radians}$$
$$= 90° 2'{\cdot}9.$$

Examples illustrating the use of Integral forms. By R. HARGREAVES, M.A., St John's College.

[*Received* 16 July 1915.]

The following are examples of the methods described in a paper on "Integral forms and their connexion with Physical Equations*." The first deals with the invariance of the electromagnetic equations under a transformation of time and one coordinate without assumption as to the functional character of the relations. The second introduces a characteristic or generating function for electromagnetic action by analogy with Clebsch's theorem in Hydrodynamics. The third connects Clebsch's form in Hydrodynamics with the methods of the paper.

EXAMPLE I. The integral forms to which the electromagnetic equations are related, (18) and (21) *op. cit.*, are

$$\Omega_2(e)=\int X\,dydz+Ydzdx+Zdxdy+Vadtdx+Vbdtdy+Vcdtdz \qquad \text{.....(1 a),}$$

and

$$\Omega_2(m)=\int a\,dydz+bdzdx+cdxdy-VXdtdx-VYdtdy-VZdtdz \qquad \text{.....(1 b).}$$

Any transformation which leaves the electromagnetic equations unaltered in form, must also make these forms invariant. We suppose $x=x'$, $y=y'$, but t and z functions of t' and z'; and make no assumption as to the nature of the functions. Then $\Omega_2(e)$ becomes

$$\begin{aligned}\int\Big(X\frac{\partial z}{\partial z'}-Vb\frac{\partial t}{\partial z'}\Big)dy'dz'+\Big(Y\frac{\partial z}{\partial z'}+Va\frac{\partial t}{\partial z'}\Big)dz'dx'+Zdx'dy'\\ +V\Big(a\frac{\partial t}{\partial t'}+\frac{Y}{V}\frac{\partial z}{\partial t'}\Big)dt'dx'+V\Big(b\frac{\partial t}{\partial t'}-\frac{X}{V}\frac{\partial z}{\partial t'}\Big)dt'dy'\\ +Vc\Big(\frac{\partial t}{\partial t'}\frac{\partial z}{\partial z'}-\frac{\partial t}{\partial z'}\frac{\partial z}{\partial t'}\Big)dt'dz';\end{aligned}$$

and we infer that $X'\ldots c'$ must be given by

$$\left.\begin{aligned}&X'=X\frac{\partial z}{\partial z'}-Vb\frac{\partial t}{\partial z'},\quad Y'=Y\frac{\partial z}{\partial z'}+Va\frac{\partial t}{\partial z'},\quad Z'=Z,\\ &a'=a\frac{\partial t}{\partial t'}+\frac{Y}{V}\frac{\partial z}{\partial t'},\quad b'=b\frac{\partial t}{\partial t'}-\frac{X}{V}\frac{\partial z}{\partial t'},\quad c'=c\Big(\frac{\partial t}{\partial t'}\frac{\partial z}{\partial z'}-\frac{\partial t}{\partial z'}\frac{\partial z}{\partial t'}\Big)\end{aligned}\right\} \qquad \text{.....(2 a).}$$

* *Trans. Camb. Phil. Soc.* Vol. XXI. iii. 1908.

To transform (1 b), a and X must be interchanged and the sign of V altered; a second determination of $X' \ldots c'$ is reached, viz.,

$$\left.\begin{aligned} &a' = a\frac{\partial z}{\partial z'} + VY\frac{\partial t}{\partial z'}, \quad b' = b\frac{\partial z}{\partial z'} - VX\frac{\partial t}{\partial z'}, \quad c' = c, \\ &X' = X\frac{\partial t}{\partial t'} - \frac{b}{V}\frac{\partial z}{\partial t'}, \quad Y' = Y\frac{\partial t}{\partial t'} + \frac{a}{V}\frac{\partial z}{\partial t'}, \quad X' = X\left(\frac{\partial t}{\partial t'}\frac{\partial z}{\partial z'} - \frac{\partial t}{\partial z'}\frac{\partial z}{\partial t'}\right) \end{aligned}\right\} \quad \ldots\ldots\ldots(2\,\text{b}),$$

which should be in agreement with the first. The agreement is realized, the first and second lines of (2 a) become identical with the second and first lines respectively of (2 b), if

$$\frac{\partial t}{\partial t'} = \frac{\partial z}{\partial z'}, \quad \frac{\partial t}{\partial z'} = \frac{1}{V^2}\frac{\partial z}{\partial t'}, \quad \frac{\partial t}{\partial t'}\frac{\partial z}{\partial z'} - \frac{\partial t}{\partial z'}\frac{\partial z}{\partial t'} = 1 \quad \ldots\ldots\ldots(3).$$

These are the general conditions defining the relation between (t, z) and (t', z') when the forms (1 a) and (1 b) are invariant. The first condition gives $t = \dfrac{\partial \phi}{\partial z'}$, $z = \dfrac{\partial \phi}{\partial t'}$, where ϕ is a function of (t', z'). The second condition gives $\dfrac{\partial^2 \phi}{\partial z'^2} = \dfrac{1}{V^2}\dfrac{\partial^2 \phi}{\partial t'^2}$, from which we infer that

$$\phi = f(z' + Vt') + F(z' - Vt').$$

The third condition is represented by

$$1 + 4V^2 f''(z' + Vt')\, F''(z' - Vt') = 0.$$

It is not admissible to suppose that any relation exists between t' and z', and we are therefore compelled to make f'' and F'' each constant. Thus

$$\phi = \frac{1}{4V}\left\{\lambda (z' + Vt')^2 - \frac{1}{\lambda}(z' - Vt')^2\right\} \quad \ldots\ldots\ldots(4)$$

with λ constant is the only admissible solution. Seeing that for t and z we are limited to linear functions we may also write

$$\left.\begin{aligned} &z = \alpha z' + \beta t', \quad t = \gamma t' + \delta z'; \\ &\alpha = \gamma, \quad \delta = \beta/V^2, \quad \alpha\gamma - \beta\delta = 1; \end{aligned}\right\} \quad \ldots\ldots\ldots(5\,\text{a})$$

and then

or now

$$z = \gamma z' + \beta t', \quad t = \gamma t' + \frac{\beta z'}{V^2}, \text{ with } \gamma^2 - \beta^2/V^2 = 1 \ldots(5\,\text{b}).$$

Whether in this form or that of (4) we are left with the choice of one significant quantity, a constant, in terms of which the transformation is to be stated. If $w = \beta/\gamma$ is taken for that

quantity, w is a velocity of translation, and the full specification of the transformation is

$$\left.\begin{aligned} &x = x',\ y = y',\ z = \gamma(z' + wt'),\ t = \gamma\left(t' + \frac{wz'}{V^2}\right),\ \gamma^2\left(1 - \frac{w^2}{V^2}\right) = 1 \\ &X' = \gamma\left(X - \frac{wb}{V}\right),\quad Y' = \gamma\left(Y + \frac{wa}{V}\right),\quad Z' = Z \\ &a' = \gamma\left(a + \frac{wY}{V}\right),\quad b' = \gamma\left(b - \frac{wX}{V}\right),\quad c' = c \end{aligned}\right\} \quad \ldots\ldots\ldots(6).$$

The value of λ in (4) is $\sqrt{(V+w)/(V-w)}$, and

$$\phi = \frac{\gamma}{4V}\left\{\left(1 + \frac{w}{V}\right)(z' + Vt')^2 - \left(1 - \frac{w}{V}\right)(z' - Vt')^2\right\};$$

but when the conclusion that only a linear transformation is possible, is once reached, there is no point in using the function ϕ.

The form (1b) may be derived from a linear one containing vector and scalar potentials, viz.

$$\Omega_1(m) = \int F dx + G dy + H dz - V\psi dt \quad \ldots\ldots\ldots(7).$$

Its invariance under the transformation (6) yields

$$F' = F,\ G' = G,\ H' = \gamma\left(H' - \frac{w\psi}{V}\right),\ \psi' = \gamma\left(\psi - \frac{wH}{V}\right) \ldots(8).$$

The transformation of the derivative of (1a) leads to further results. This form is

$$\int \rho dx dy dz - \rho u_r dt dy dz - \rho v_r dt dz dx - \rho(w + w_r) dt dx dy$$

if the relative velocities, where density is ρ, are $u_r v_r w_r$; which on transformation becomes

$$\int \rho\left(\frac{1}{\gamma} - \gamma w w_r\right) dx' dy' dz' - \rho u_r dt' dy' dz' - \rho v_r dt' dz' dx' - \gamma\rho w_r dt' dx' dy'.$$

The invariance demands

$$\rho' = \rho\left(\frac{1}{\gamma} - \gamma w w_r\right),\ \rho' u_r' = \rho u_r,\ \rho' v_r' = \rho v_r,\ \rho' w_r' = \gamma\rho w_r \ldots(9\,a),$$

and therefore

$$u_r' = \frac{\gamma u_r}{1 - \frac{\gamma^2 w w_r}{V^2}},\quad v_r' = \frac{\gamma v_r}{1 - \frac{\gamma^2 w w_r}{V^2}},\quad w_r' = \frac{\gamma^2 w_r}{1 - \frac{\gamma^2 w w_r}{V^2}} \ldots(9\,b);$$

defining a correspondence between the components of relative velocity as expressed in the two schemes.

If we transform in the reverse direction using

$$z' = \gamma(z - wt), \quad t' = \gamma\left(t - \frac{wz}{V^2}\right),$$

$$\int \rho' dx' dy' dz' - \rho' u_r' dt' dy' dz' - \rho' v_r' dt' dz' dx' - \rho' w_r' dt' dx' dy'$$

becomes

$$\int \gamma\rho' \left(1 + \frac{ww_r'}{V^2}\right) dx dy dz - \rho' u_r' dt dy dz - \rho' v_r' dt dz dx - \gamma\rho' (w + w_r') dt dx dy,$$

and so

$$\rho = \gamma\rho' \left(1 + \frac{ww_r'}{V^2}\right), \quad \rho u_r = \rho' u_r', \quad \rho v_r = \rho' v_r',$$

$$\rho (w + w_r) = \gamma\rho' (w + w_r') \ldots (10\,\text{a}),$$

and therefore

$$u_r = \frac{u_r'}{\gamma\left(1 + \frac{ww_r}{V^2}\right)}, \quad v_r = \frac{v_r'}{\gamma\left(1 + \frac{ww_r}{V^2}\right)}, \quad w + w_r = \frac{w + w_r'}{1 + \frac{ww_r'}{V^2}},$$

or

$$w_r = \frac{w_r'}{\gamma^2\left(1 + \frac{ww_r'}{V^2}\right)} \quad \ldots\ldots\ldots\ldots\ldots\ldots (10\,\text{b}).$$

The equations (10) are an inversion of (9) through the formula

$$\left(1 + \frac{ww_r'}{V^2}\right)\left(1 - \frac{\gamma^2 ww_r}{V^2}\right) = 1.$$

The geometrical relations of (6) when we form the differentials and write

$$dz = (w + w_r)\, dt, \quad dx = u_r dt, \ldots, dz' = w_r' dt'$$

also imply a correspondence between the two expressions of relative velocity; which will be found to agree with (9b) or (10b) according as we pass from plain to dashed letters or *vice versâ*. The agreement is of course a necessary part of the consistency of the scheme of transformation.

Example II. *An Electromagnetic Characteristic or Generating function.*

The electric and magnetic vectors are given in (1b) which is derivable from (7) by the reticular operation. When a single source is in question, (7) can be expressed as $\int \lambda d\mu + d\phi$, and ϕ

will be ineffective. If for a point-charge e_1 a disturbance emitted at time and place $(t_1 x_1 y_1 z_1)$ reaches a place (xyz) where the vectors are sought at time t, then with

$$r_1^2 \equiv \Sigma (x - x_1)^2, \quad t_1 = t - r_1/V,$$

and

$$Vs \equiv Vr_1 - \Sigma (x - x_1) u_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11),$$

$\Omega_1 (m)$ is defined as

$$\frac{e_1}{V} \int (u_1 dx + v_1 dy + w_1 dz - V^2 dt)/s.$$

But
$$\begin{aligned} Vds &= Vdr_1 - \Sigma u_1 dx + \Sigma u_1^2 dt_1 - \Sigma (x - x_1)\, \dot{u}_1 dt_1 \\ &= V^2 dt - \Sigma u_1 dx - \{ V^2 - \Sigma u_1^2 + \Sigma (x - x_1)\, \dot{u}_1 \}\, dt_1, \end{aligned}$$

and therefore

$$\Omega_1 (m) = - e_1 \int \frac{ds}{s} - e_1 \int \frac{V^2 - \Sigma u_1^2 + \Sigma (x - x_1)\, \dot{u}_1}{Vs} dt_1;$$

the effective section of which is

$$\Omega_1 (m) = - e_1 \int \frac{V^2 - \Sigma u_1^2 + \Sigma (x - x_1)\, \dot{u}_1}{Vr_1 - \Sigma (x - x_1)\, u_1} dt_1 \equiv - Ve_1 \int \chi_1 dt_1 \ldots (12).$$

Forming the derivative $\Omega_2 (m)$ we have

$$X = e_1 \left(\frac{\partial t_1}{\partial x} \frac{\partial \chi_1}{\partial t} - \frac{\partial t_1}{\partial t} \frac{\partial \chi_1}{\partial x} \right), \ldots, \quad a = Ve_1 \left(\frac{\partial t_1}{\partial y} \frac{\partial \chi_1}{\partial z} - \frac{\partial t_1}{\partial z} \frac{\partial \chi_1}{\partial y} \right), \ldots,$$

$\ldots\ldots\ldots (13)$

as values of the vectors. For the force on a charge e_2 at $(t x_2 y_2 z_2)$, viz., $\xi_2 = e_2 \left\{ X + \dfrac{v_2 c - w_2 b}{V} \right\}$, we get

$$\xi_2 = e_1 e_2 \left(\frac{\partial t_1}{\partial x_2} \frac{d\chi_1}{dt} - \frac{dt_1}{dt} \frac{\partial \chi_1}{\partial x_2} \right),$$

and

$$\Sigma \xi_2 u_2 = e_1 e_2 \left(\frac{dt_1}{dt} \frac{\partial \chi_1}{\partial t} - \frac{\partial t_1}{\partial t} \frac{d\chi_1}{dt} \right) \ldots\ldots\ldots\ldots\ldots (14),$$

where

$$\frac{d}{dt} = \frac{\partial}{\partial t} + u_2 \frac{\partial}{\partial x_2} + v_2 \frac{\partial}{\partial y_2} + w_2 \frac{\partial}{\partial z_2}.$$

This force is derivable from a kinetic potential $- e_1 e_2 \chi_1 \dfrac{dt_1}{dt}$, as is readily verified. The reason this modification of kinetic potential is possible is that $Fu_2 + Gv_2 + Hw_2 - V\psi$ differs from $- Ve_1 \chi_1 \dfrac{dt_1}{dt}$ by a quantity $- \dfrac{e_1}{s} \dfrac{ds}{dt}$, which being a complete time-rate of a function yields no force.

It should be stated that though formulae (13) and (14) lead to the usual results, there is no gain but rather loss in facility of reaching them. But the form used is striking, and the failure to apply it in any advantageous way may well be due to my own shortcomings.

EXAMPLE III. *Clebsch's form in Hydrodynamics.*

The integral form used p. 116 *op. cit.* was

$$\Omega = \int u dx + v dy + w dz + T dt,$$

in which T was found to be $-f - \frac{1}{2}\Sigma u^2$, the hydrodynamical equations being

$$\frac{Du}{Dt} = -\frac{\partial f}{\partial x}, \ \ldots, \text{ i.e. } f = \frac{p}{\rho} + V \text{ or } \int \frac{dp}{\rho} + V.$$

If we quote the pressure equation in the form

$$f + \tfrac{1}{2}\Sigma u^2 + \frac{\partial \phi}{\partial t} + \lambda \frac{\partial \mu}{\partial t} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (15)$$

(cf. Basset's *Hydrodynamics*), then obviously

$$T = \frac{\partial \phi}{\partial t} + \lambda \frac{\partial \mu}{\partial t}, \text{ as well as } u = \frac{\partial \phi}{\partial x} + \lambda \frac{\partial \mu}{\partial x}, \ \ldots, \ldots \ \ldots (16).$$

Thus Ω is identified in respect to all terms with $\int \lambda d\mu + d\phi$.

The resolution of asymmetric quinquevalent nitrogen compounds. By Jos. Reilly (M.A., M.Sc., 1851 Exhibition Scholar), Emmanuel College. (Communicated by Professor Pope.)

[*Read* 25 October 1915.]

Many peculiarities have been recorded with regard to the varying facilities with which different optically active acids can be utilised to effect the separation of asymmetric quinquevalent nitrogen compounds into their optically active constituents. In the first successful resolution of a compound of this class, namely benzylphenylallylmethylammonium iodide, Pope and Peachey (*Trans. Chem. Soc.* 1899, lxxv. 1127) fractionally crystallised the compound obtained by replacing the iodine by the *d*- or *l*-β-camphorsulphonate radical, yet Wedekind (*Ber.* 1899, xxxii. 517) previously had failed to obtain the two modifications by repeated crystallisation of the *d*-tartrate or *d*-camphoric acid derivative. Later, Jones failed to separate dextro- and laevo-phenylmethylethylallylammonium *d*-β-camphorsulphonate into its two constituents even after repeated crystallisation from various solvents, while he found that the dextro- or laevo-α-bromocamphor-π-sulphonate salt of the inactive base could readily be resolved by fractional crystallisation.

It might therefore be considered that the stronger the optically active acid employed to effect the resolution the more easily would such a result be achieved. This, however, is not always the case as the following results show. Wedekind (*Zeit. physikal Chem.* 1903, xlv. 235), aware of the previous work of Pope and his pupils, attempted to resolve *p*-tolylbenzylmethylallylammonium iodide into its two antipodes by the aid of *d*-β-camphorsulphonic acid, but failed, yet Jones (*Trans. Chem. Soc.* 1908, xciii. 1790, cf. Homer, *Proc. Camb. Phil. Soc.* 1907, xiv. ii. 196) succeeded by fractional crystallisation of the hydrogen *d*-tartrate. Several other instances have been recorded in which resolution is brought about by the aid of the comparatively weak acids, *d*- or *l*-tartaric or *d*- or *l*-camphoric, where the much stronger acids such as *d*- or *l*-β-camphor-sulphonic, or *d*- or *l*-α-bromocamphor-π-sulphonic have been unsuccessful.

Racemisation effects have been advanced as the cause of non-resolution in many cases. A similar explanation has been put forward to explain the failure to separate into dextro- and

laevo-modifications certain cyclic ammonium compounds such as the α- and β-substituted pyridinium compounds or the tetrahydroquinoline derivatives of the type $C_9H_{10}NR'R''X$. These compounds should be capable of existing as optical antipodes since their molecules have no plane of symmetry (Jones, *Trans. Chem. Soc.* 1903, LXXXIII. 1400). In other cases it is thought that hydrolytic dissociation of the salts will account for the results obtained. In the particular compounds studied in the present work neither of these theories has been found sufficient. Similar solubilities of the two derivatives formed from the dextro- and laevo-base and the optically active acid would seem to be a simpler explanation. This selective action of optically active acids, in their power to bring about the resolution of externally compensated compounds, noticeable much more frequently in the case of nitrogen than of carbon derivatives, would suggest that the non-resolution in particular cases is due to some other causes than those indicated.

The present work was undertaken in order to investigate more fully this selective action of optically active acids. The original quaternary iodide resolved by Pope and Peachey (*Trans. Chem. Soc.* ibid.) is readily obtained in the dextro- and laevo-forms by fractional crystallisation of the *d*- and *l*-β-camphorsulphonates of the inactive ammonium base and reformation of the iodides by the action of aqueous potassium iodide. If α-bromocamphor-π-sulphonic acid is substituted for β-camphorsulphonic acid in the above experiments, a similar separation is much more difficult to obtain on fractional crystallisation of the α-bromocamphor-π-sulphonate. To ascertain if racemisation sufficiently accounts for the different behaviour of the β-camphorsulphonates and the halogenated compounds, an attempt was made to obtain the pure dextro- and laevo-compounds of the base with dextro- and laevo-α-bromocamphor-π-sulphonic acid by an indirect method and so to test the hypothesis by actual experiment.

By the reaction of anhydrous silver *d*-α-bromocamphor-π-sulphonate with *d*-benzylphenylallylmethylammonium iodide in dry acetone under certain conditions the compound, *d*-benzylphenylallylmethylammonium *d*-α-bromocamphor-π-sulphonate, was obtained in a pure form.

By combination of the dextro- and laevo-iodide with silver dextro- and laevo-α-bromocamphor-π-sulphonate under different conditions the following compounds were obtained: *dBdA*, *dBlA*, *lBlA* and *lBdA*.

Preliminary study of these compounds has shown that they are of the same order of stability as the corresponding β-camphorsulphonates and show only a very slight tendency to racemise.

A solution of 0·1655 grams of *d*-benzylphenylallylmethylam-

monium *d*-α-bromocamphor-π-sulphonate (melting at 155–159° C. with decomposition) in water at 14° C., gave $[\alpha]_D = 81{\cdot}5°$. After two months in a thermostat at 20° C., the same solution gave $[\alpha]_D = 79{\cdot}9°$. On boiling the solution, however, it became cloudy, oily globules making their appearance. The pure compound can be recrystallised from dry ethyl acetate without any racemisation occurring. On recrystallisation from alcohol a slight fall in the rotatory power is observed, while on crystallisation from chloroform solution the fall is more marked. Further study of the properties of these compounds is in progress.

The above investigation was undertaken on the suggestion of Professor Pope, to whom the thanks of the author are due both for the suggestion and for his continued interest during the work.

On a little-known concealed coalfield in Oxfordshire. By E. A. Newell Arber, Sc.D., F.G.S., Trinity College.

[*Read* 22 November 1915.]

At the present time the existence of only two wholly concealed coalfields has been proved in this country. Both of these lie in the very large hidden area to the South of the Midland fields, and to the East of the exposed or partly exposed fields of the West of England. This district, including the Home and Southern Counties and East Anglia, may be conveniently subdivided, though in a purely arbitrary manner, into two nearly equal halves by a line joining Bristol and London. In the more Southern section lies the Kent coalfield, which is still the only wholly concealed Coal Measure area of which we have any real knowledge. It was first proved in 1890.

The fact however is often overlooked that the first entirely concealed field discovered in England lies in part at least in Oxfordshire, in the Northern section of our district. Coal Measures were here proved in or before 1877, in a boring at Burford Signet near Witney. It is true however that, with the possible exception of another boring at Lower Lemington near Batsford, the details of which have only recently been made known to us, no progress has since been made in the matter of probing the nature, extent and resources of the Oxfordshire field.

While unfortunately no new facts are available for publication here in regard to this coalfield, it may be worth while to reconsider briefly the present evidence, in the light of the better knowledge which we now possess of the adjoining coalfields.

The accounts which have been published of the Burford Signet Coal Measures are neither so explicit nor as detailed as one could wish. Situated a little to the South of Burford, and to the West of Witney in the S.W. corner of Oxfordshire, not far from the Gloucestershire border, the boring in question was put down between 1875 and 1877. The Mesozoic cover of Jurassic and Triassic rocks was only 1184 ft. in thickness. The O.D. of the boring was 350 ft. Translating the record with reference to this constant, we find that the Coal Measures were struck at 834 ft. O.D. and that 226 ft. of the same rocks were penetrated*.

* De Rance, C. E., *Trans. Manchester Geol. Soc.*, Vol. xiv. p. 437, 1878; *Rep. Brit. Ass. Dublin* (1878), p. 382, 1879; see also Woodward, H. B., Jurassic Rocks of Britain, Vol. iv., *Mem. Geol. Surv.*, 1894, p. 303.

The boring ended in Coal Measures. These are described as alternations of red and grey sediments, dark shales alternating with red marls, and pale or red or green sandstones. Only one seam of coal was proved, the thickness of which was not stated. It was however probably quite thin.

There is only one other boring at present, which may throw any light on the results obtained at Burford. Between 1901—4 a boring was put down near Batsford, or more correctly at Lower Lemington in Gloucestershire. The full details of these cores were first published in 1913*. Here 1021 ft. of Jurassic and Triassic cover were penetrated, the O.D. being 380 ft. The Coal Measures were struck at 641 ft. O.D., and the boring passed into Silurian rocks at 1546 ft. from the surface (1166 ft. O.D.). The Coal Measures were thus only 524½ ft. thick.

The measures are described as sandstones and conglomerates, with grey and red shales and red marls. No coals were proved, but coal veins occur in the arenaceous beds. These measures have been termed Upper Coal Measures, and one bed of sandstone is described as "resembling Pennant." In addition to specimens of *Anthracomya* and Ostracoda, a small flora was obtained from the measures, which will be further discussed here.

It is by no means certain that the measures penetrated at Burford and Batsford belong to the same coalfield, though I am inclined, on the whole, to make this assumption. The distance between the two borings is about 15 miles, along an almost due North and South line, and, so far as this matter is concerned, there is nothing against the view that the measures proved in the two borings belong to the same field, seeing that the major axes of the neighbouring fields all trend more or less North and South. There is also a very strong lithological similarity between the rocks proved at each boring. They appear to consist of *red-grey* sediments, the presence of red clays or marls being, as we shall see, remarkable.

The field proved at Burford may be conveniently distinguished as the Oxfordshire Coalfield. No other concealed field is of course known at present from this county, whereas several distinct coalfields occur in Gloucestershire. It is also probable that it may eventually be found to transgress into Gloucestershire, Warwickshire and Worcestershire. In the neighbourhood of Batsford, which is actually in Gloucestershire, county boundaries become very intricate, no less than four shires being involved. As is well known the application of county names to coalfields is frequently misleading and unwise, since the fields often overflow into neighbouring counties. For this reason it may eventually be

* Strahan, A., Batsford (or Lower Lemington) boring, near Moreton-in-Marsh, *Summ. Progr. Geol. Surv.* for 1912, p. 90, 1913.

necessary to abandon the provisional suggestion as to the name employed here. At the present stage of our knowledge, however, the term Oxfordshire Coalfield seems to be the simplest solution of the difficulty.

Assuming that we are dealing in the case of these two borings with one and the same coalfield, the next point to consider is the evidence of the palaeobotanical horizon. No plant remains appear to have been found at Burford, nor are any recorded. The small flora recorded from Batsford is unfortunately not sufficiently large to be quite conclusive as to the horizon. However there is no doubt that it must belong to either the Upper, or the Transition Coal Measures. The most important and abundant species are *Pecopteris Miltoni* (Art.), *Neuropteris Scheuchzeri* Hoffm., *N. rarinervis* Bunb., and *Cordaites principalis* (Germ.). The last is not an Upper Coal Measure plant. All the other species are commonly associated in Kent as a Transition Coal Measure assemblage. Two other Neuropterids (*N. flexuosa* Sternb. and *N. ovata* Hoffm.) are also frequent on this horizon, though, like the others, not confined to it. Thus, while admitting some uncertainty as to the horizon, I am impressed with the possibility of it being Transition Coal Measures.

From this reconsideration of the evidence I arrive at the provisional conclusion that, in the Oxfordshire field, there are probably Transition Coal Measures of a red-grey facies.

These measures at Batsford overlie Silurian rocks, and are probably near an outcrop, which may or may not be the Northern outcrop of the field. It is therefore probable that neither the Middle nor the Lower Coal Measures are represented in this area, unless there be some purely local peculiarity of the field in this neighbourhood. The beds at Burford much further South, provided they belong to the same field, will thus prove to belong to the same, or to an even higher horizon than at Batsford.

I come now to a comparison with the results which have recently been obtained in the coalfields which lie to the West and Northwest of the Oxfordshire field. A line joining Burford and Batsford, and produced northwards, would run in the concealed ground between the South Staffordshire and Warwickshire coalfields. With these areas however there is no real comparison. It is true that, in both fields, red-grey measures belonging to the Transition Series occur, but these are underlain by productive Middle Coal Measures, which appear to be entirely wanting in the Oxfordshire field. These three fields however agree in the absence of Lower Coal Measures, and in the fact that the measures rest directly on Silurian rocks in South Staffordshire and Oxfordshire, though on Cambrian in Warwickshire.

With the most southerly of the coalfields of the Welsh

Borderland, the comparison is closer. The southern field of the Wyre Forest-Coalbrookdale area* consists entirely of red-grey Transition Coal Measures, resting, not it is true on Silurians, but on Old Red Sandstone. Still further South, and due West of the Burford-Batsford line, we have the little known and almost entirely concealed field of Newent†, near Gloucester, where exactly the same phenomena are observed.

With the fields lying to the Southwest of Batsford, there is no comparison. No red-grey beds occur in the Forest of Dean, where the whole of the measures belong to the Upper Coal Measures‡. In the Somerset field, with its Bristol outlier, Transition Coal Measures occur, but the lithological facies of the rocks is entirely different. The only red beds known to me here are the red shales, between the Radstock and Farrington Series, of 130—250 ft. in thickness. The horizon of these beds is Upper Coal Measures.

Thus the closest comparison with the Oxfordshire field appears to me to be with the Wyre Forest and with Newent, both as regards horizon and in respect to the lithology, and, so far as our present knowledge extends, it looks as if this concealed area would eventually prove to be of the same type. It is much to be hoped that further explorations will be made in this field. At present it must be confessed that the economic prospects of the area do not appear to be very promising. It cannot however be claimed that these two borings have done more than to prove the existence of the field, the real resources of which remain quite unknown. It must be further borne in mind that the red-grey Transitions of the two neighbouring fields above mentioned are both productive, at least *along their Western margins.* The Wyre Forest field is still being actively worked. The Newent area has been worked at various periods, though the extent and resources of this almost entirely hidden field have never been ascertained. These facts are at least in favour of a more thorough exploration of the Oxfordshire field.

* Arber, *Phil. Trans. Roy. Soc.*, Ser. B, Vol. 204, p. 363, 1914.
† Arber, *Geol. Mag.*, Dec. v. Vol. VII. p. 241, 1910.
‡ Arber, *Phil. Trans. Roy. Soc.*, Ser. B, Vol. 202, p. 233, 1912.

Notes on Certain Protozoa which may be found in cases of Dysentery from the Mediterranean War Zone. By H. B. FANTHAM, D.Sc. Lond., M.A., Christ's College, Cambridge, and Liverpool School of Tropical Medicine, and ANNIE PORTER, D.Sc. Lond., Beit Memorial Research Fellow, Quick Laboratory, Cambridge.

[*Read* 22 November 1915.]

At the present time, when the conservation of life is so important, it may be well to give a short account of some of the protozoal organisms associated with dysentery; more especially as the literature relating to many of these parasites is scattered or in relatively little-known journals, some of which are published in South America. These notes are based on personal knowledge and examination of cases from the Mediterranean regions during the present war, and are presented in the hope that they may be of service to those having charge of dysenteric cases from those parts.

Briefly, the chief types of dysentery may be classified as bacillary and protozoal. The former cannot be dealt with here. Dysenteries of protozoal origin may be grouped as amoebic, flagellate and ciliate. Cases of multiple infections of these parasites with each other or with bacilli causing dysentery may be encountered.

Amoebic Dysentery.

The best known of the protozoal dysenteries is that due to *Entamoeba histolytica*. This organism is polymorphic. The life-cycle, as now accepted, has been worked out more particularly by Darling, and James and Deeks in the Panama Canal zone, while its successful treatment by emetine has been brought forward chiefly by Rogers.

Entamoeba histolytica is found in freshly voided stools that are usually blood-stained and contain strings of mucus. The entamoebae when active show pseudopodia, at first chiefly composed of ectoplasm. The endoplasm usually contains a number of red blood corpuscles and other débris. The nucleus of the form of this organism that used to be termed *E. tetragena* may show a karyosome and a centriole.

The Entamoeba multiplies by binary fission and also by schizogony, four merozoites being produced. Encystment occurs, and the round cysts finally produced measure 12μ to 15μ in

diameter. The mature cysts contain four nuclei, as well as darkly staining masses of various shapes known as chromidial or crystalloidal blocks. The tetranucleate cyst is characteristic and diagnostic, and the cysts are the infective stages.

A patient showing acute symptoms of dysentery is not wholly infective, for he is often merely harbouring the large trophozoites of *E. histolytica* which, by animal experiments, have been shown usually to be non-infective when fed by the mouth. The stools of convalescent and recovered patients may still contain cysts, and so such persons may act as carriers of the disease. In return cases, or cases that have remained without treatment for a long time, a generation of smaller trophozoites is associated with or replaces the larger ones. These smaller forms are the senile or pre-cyst generation of Darling. These small entamoebae were described separately by Elmassian in 1909 as *Entamoeba minuta*, from a case of chronic dysentery in Paraguay.

Follicular abscesses followed by ulcers, due to *E. histolytica*, occur in the large intestine. The entamoebae may invade the liver, producing abscesses in that organ.

In the stools of sub-acute cases of amoebic dysentery which we have examined, uninucleate or tetranucleate cysts were observed.

The dysentery-producing amoebae must be distinguished from *Entamoeba coli*, a parasite which may be found in the alimentary tracts of healthy persons. *E. coli* divides by binary fission or by schizogony into eight daughter forms. The cysts contain eight nuclei when mature. They measure about 15μ to 20μ in diameter, and so are slightly larger than those of *E. histolytica*. The cyst wall of *E. coli* is thicker than that of *E. histolytica*, and the former cysts rarely contain chromidial blocks, while the trophozoites very rarely ingest red blood corpuscles.

Flagellate Diarrhoea and Dysentery.

The examination of the stools of a number of patients from Gallipoli, who showed dysenteric symptoms, revealed the presence of various flagellates. These Mastigophora include *Trichomonas hominis* (also called *T. intestinalis*), *Chilomastix* (*Tetramitus*) *mesnili*, *Giardia* (*Lamblia*) *intestinalis*, and *Cercomonas hominis*. Most of these organisms are either complicated in structure or relatively little known. While some workers may consider some of these flagellates to be non-pathogenic, others have found them to be excitants of diarrhoea and dysentery. Among these latter workers are some of the principal parasitologists of South America and the French Colonies, while their publications are of recent date.

Trichomonas hominis (or *T. intestinalis*) as found in the intestine is pear-shaped, with three free flagella at the blunt or anterior end, a lateral flagellum attached to the body by an undulating membrane and an axial rod running towards the pointed end of the body from near the anteriorly placed nucleus. The flagellate measures about 10μ to 15μ by 5μ. Rounded, contracted forms may be found in the faeces, the occurrence of encysted forms being disputed. Similar trichomonads occur in rodents such as rats and mice. Possibly these act as reservoirs of the parasites. There is also reason to believe that the trichomonads may be water-borne. Regarding pathogenicity, it may be remarked that, among recent workers, Mello-Leitao (1913) has found *T. hominis* in relatively benign dysentery in children in Rio de Janeiro. Escomel (1913) found 152 cases of dysentery in Peru due solely to *Trichomonas*. In some of the dysenteric cases invalided from Gallipoli, it has been found that the *Trichomonas* is coexistent with one or other of the dysentery bacilli. Such mixed infections have led certain workers at home to consider trichomonads as of little pathogenic importance, but others do not hold this opinion.

Chilomastix (*Tetramitus*) *mesnili*. This flagellate, first described from human faeces in 1910, is allied to *Trichomonas*, but *Chilomastix mesnili* has a large cytostome, hence its former name, *Macrostoma mesnili*. Three anterior flagella are present, and a fourth one (perhaps attached to an undulating membrane) occurs in the cytostome or oral groove, and vibrates therein. An axial rod or axostyle is absent in the organisms, which are pear-shaped, with vacuolated cytoplasm, and may measure 14μ by 7μ. Brumpt (1912) considers *Chilomastix* to be the causal agent of a colitis, but Nattan-Larrier hardly shares his opinion. Such flagellates have been found by us in the fresh faeces of patients from the Mediterranean.

Giardia (*Lamblia*) *intestinalis* is a somewhat complicated flagellate protozoön exhibiting bilateral symmetry. Eight flagella, arranged in four pairs, are present. The organism also contains two axostyles, and two nuclei, each with a karyosome, are present. On the under surface there is a concave depression or sucking disc anteriorly. The organism is from 10μ to 21μ long and 5μ to 12μ broad.

The parasite occurs in the small intestine of man, and closely allied forms, probably only varieties of *Giardia intestinalis*, may be found in rats, mice, rabbits and guinea-pigs. The flagellates attach themselves by their sucking discs to the epithelial cells of the duodenum and other parts of the small intestine of their host. They do not appear to multiply as flagellates, but division occurs within the resistant cysts that are produced. The rounded

or oval cysts, containing four nuclei and the remains of the axostyles, may be found in the faeces of diarrhoeic patients. In Indo-China, many cases of lamblial dysentery have been described by French investigators. In 1879 Grassi observed this flagellate in mice and subsequently in man in Italy. By experiment on himself he proved that infection took place by ingestion of the cysts. Cereal foodstuffs, contaminated with Lamblia cysts from the vermin of the locality, such as rats and mice, serve to convey the infection to man. Mathis (1914) in Tonkin found infected rats and mice in the houses of patients. He also discovered healthy carriers of Lamblia cysts.

The much-discussed *Cercomonas hominis* has also been found by us in the stools of dysenteric patients from Gallipoli.

Notes on Geographical Distribution and Recent Treatment of Flagellate Dysenteries.

Escomel (1913) recommends the use of essence of turpentine for *Trichomonas* dysentery. Derrien and Raynaud (1914) found this treatment effective in cases in Algeria. Other cases have occurred in the Southern United States and were treated with thymol and calomel at night, followed by Carlsbad salts in the morning. Escomel, Smithies and others have found the flagellates in water reservoirs and in unfiltered surface waters which were in use for drinking purposes. In Peru, after the cleaning of the reservoirs and consequent removal of the flagellates, the cases of dysentery ceased. As Stiles points out, when flagellates or amoebae are found in 10 to 40 per cent. or more of the members of a community, means should be taken to improve the methods for the disposal of dejecta in order to safeguard the food supply against faecal contamination. Such contamination may occur by the agency of rodents, flies, and water.

Dysentery due to the allied flagellate, *Chilomastix* (*Tetramitus*) *mesnili*, has been recorded within the last three years by Brumpt from France, by Nattan-Larrier from the Ivory Coast, by Marques da Cunha and Torres from Brazil, by Gäbel from Tunis, and we have seen cases from Gallipoli. Methylene blue has been recommended for the treatment of such cases.

Giardia (*Lamblia*) *intestinalis* has been studied recently in Tonkin by Mathis (1914), who pointed out that the patients' homes were infested with rats and mice. According to Noc, Lamblia may be water-borne, while healthy carriers of cysts are known. Emetine hydrochloride may be useful in killing the flagellate forms, but apparently has little action on the cysts. Magnesium sulphate has been recommended, also milk diet with

calomel treatment. Uzara tablets and extract of male fern have been recommended by certain investigators.

Spirochaetes, some of them resembling *S. eurygyrata*, Werner, were found in the faeces of certain patients.

Ciliate Dysentery.

Balantidium coli is the chief causal agent of ciliate dysentery. The parasite is relatively large, with an oval body, 60μ to 100μ (or even 200μ) long by 50μ to 70μ broad There is a funnel-shaped cytostome at one pole. The organism has a macronucleus and a micronucleus, and two contractile vacuoles. A cytopyge is also present. Occasionally, ingested blood corpuscles are found in the endoplasm. The parasites form round cysts.

A smaller species, *Balantidium minutum*, is also known.

Balantidia occur in the large intestine of man and in the rectum of the domestic pig. The parasites are able to penetrate the intestinal walls of man and give rise to ulcers, though these are rare in pigs. Epidemics have also been recorded in monkeys.

Cases of balantidiasis occur especially among swineherds, and farm hands dealing with pigs, pork butchers and persons engaged in similar occupations. Personal cleanliness of such people is, then, of the greatest importance, while pigs should be confined and not allowed to run in yards and dwellings. As swineherding is an important occupation in Serbia, the possibility of balantidial dysentery among troops operating in that country must be borne in mind. The disease is widely distributed in various parts of the world.

Treatment. The use of thymol has yielded good results, and some have recommended the further use of de-emetinised ipecacuanha to destroy any persistent balantidia. Enemata of collargol or protargol may be tried. The direct effect of emetine is disputed.

On Induced Herpetomoniasis in Birds. By H. B. FANTHAM, D.Sc. Lond., M.A., Christ's College, Cambridge, and Liverpool School of Tropical Medicine, and ANNIE PORTER, D.Sc. Lond., Beit Memorial Research Fellow, Quick Laboratory, Cambridge.

[*Read* 22 November 1915.]

Contents.

Introduction.

Recently, we have published the results of a number of our experiments, extending over some years, on the introduction of certain insect flagellates into various vertebrates, belonging to the Pisces, Amphibia, Reptilia and Mammalia. For some time past, we have been testing the pathogenicity of certain insect flagellates with respect to birds, the last great group of European vertebrates that had remained untested by us The present paper records the results of introducing *Herpetomonas jaculum*, Léger, and *H. culicis*, Novy, MacNeal and Torrey, into birds. As long ago as 1907, Drs Edm. and Et. Sergent briefly recorded and figured a herpetomonad that they had found in the blood of a pigeon. This fact is of much interest. Many birds are found dead every year from unknown causes, old age and accidents being excluded. Some of our experiments, coupled with the fact that the crops of the birds often contain insect remains, suggest that undetected herpetomoniasis may be the cause.

We have much pleasure in thanking Professor G. H. F. Nuttall, F.R.S., for his kind interest in our researches, and for looking at many of our preparations.

Material and Methods.

The birds used were canaries (*Serinus canarius*), sparrows (*Passer domesticus*) and martins (*Chelidon urbica*). The insects, *Nepa cinerea* and *Culex pipiens*, were obtained chiefly from the

neighbourhood of Cambridge, and some of the Culex larvae were bred out and identified.

The birds were fed either with the entire insects containing herpetomonads, or with the alimentary canals that had been removed. There was usually no difficulty over the feeding. After the infective feed had been given, grain and shredded cooked meat or egg were given as food.

Blood smears of the experimental birds were taken at intervals, and smear preparations of the organs were made at death. Wet fixation by osmic vapour followed by absolute alcohol, or Bouin's fluid was used, and the preparations were stained with Giemsa's solution or iron haematoxylin. Control birds were kept and remained healthy.

Experimental Work.

Herpetomonas jaculum has been shown by us to be capable of infecting certain fish, amphibia, snakes and mice. It has also proved infective to birds. The use of *H. culicis* was suggested by finding the remains of many Culex in the crops of some sparrows and martins found dead and sent to us for examination. They had been too long dead to allow of the detection of herpetomonads, had any been present.

A short outline of the experiments may be given.

Experiment 1 (H.B.F.). A female canary was fed on the intestines of two *Nepa cinerea* containing *Herpetomonas jaculum* and on two infected nymphs. The bird weighed 26 grams. It died 51 days later, when it weighed 10·2 grams. At post-mortem, the body was much emaciated, the liver seemed normal, the spleen slightly enlarged and the suprarenal bodies were very hard and firm. Non-flagellate, leishmaniform parasites were found in smears of the heart, liver, spleen, lungs, kidney and bone-marrow. Elongating forms occurred in the liver and also a few typical, flagellate herpetomonads. Non-flagellate elements were more abundant than flagellate forms.

Experiment 2 (A.P.). A female sparrow was fed with a number of larvae and adults of *Culex pipiens* containing *H. culicis*. Four days after feeding, a blood smear showed a single non-flagellate herpetomonad. On the ninth day of the experiment, the bird died suddenly. The liver and spleen were softish, the bone-marrow small in quantity and more fluid than usual. The other organs were normal. Stained preparations of the organs showed that, as in the case of the canary, there was a generalised infection of the parasite. The flagellate form of *H. culicis* predominated. The heart, liver, lungs, kidneys and suprarenal bodies contained well developed flagellates. Non-flagellate forms were

present in them as well as in the bone-marrow. Dividing leishmaniform parasites were found in the heart and liver; elongating elements in process of division occurred in the bone-marrow, while fully developed flagellates in various stages of division were found in smears of the heart, liver and lungs.

Experiment 3 (H.B.F.). A young male martin was fed with larvae and mature *Culex pipiens*, containing *Herpetomonas culicis*. It lived twelve days after the infective feed. Blood smears taken during the course of the infection were negative. At post-mortem, the condition of the bird and the distribution of the parasites were found to be the same as in the case of the sparrow detailed in Experiment 2.

Experiment 4 (A.P.). A young female martin was inoculated subcutaneously with *H. culicis*. It was frightened and died after two days. No infection appeared to have taken place.

Experiment 5 (A.P.). A young male canary was fed with the faeces of *Nepa cinerea*, the excrement which contained *H. jaculum* having been collected on a slide and mixed with bread. Blood examinations were made at intervals. The bird lived 17 days after the infective feed. A few non-flagellate forms were found on the 7th and 11th days. At post-mortem, a few non-flagellate forms were found in the liver and spleen, elongating parasites in the liver and bone-marrow, and a few flagellates in the liver. The heart blood and tissue contained some multiplicative forms.

Experiment 6 (H.B.F.). A young male martin was fed with the faeces of several larvae and adults of *Culex pipiens*, mixed with small quantities of boiled mutton. The Culex faeces contained post-flagellate or encysted stages of *H. culicis*. The bird lived 32 days after the infective feed. The body was somewhat emaciated at death. Non-flagellate forms of *H. culicis* were present in the spleen and lung, elongating forms in the bone-marrow and a very few flagellate stages in the spleen.

Experiment 7 (H.B.F.). A female sparrow was fed with the faeces of *Nepa cinerea* containing *H. jaculum*. On the 11th day after the infective feed, a probable parasite of the elongating, flagellate type was seen in the blood, but no others have been seen since. The bird is growing somewhat thinner, but is still alive at the time of writing, and infection appears doubtful.

Experiment 8 (A.P.). A female canary was fed with food contaminated with *H. culicis*. As usual, blood smears were taken at intervals. No parasites were found. After 80 days, the bird was killed, but no herpetomonads were found on examination of organ smears.

In connection with these experiments, it should be remembered that the flagellates of the insect hosts rarely coexisted with many bacteria. In common with certain other workers, we found that

when the insects contained many bacteria, the protozoal flagellates usually disappeared. Further, some digestion experiments performed by us have shown that many bacteria introduced into the digestive fluids of the bird's stomach are destroyed by the same, while the flagellates are but little affected. This is not surprising, since certain protozoal infections of man are known to flourish in an acid medium, and are combated by the use of alkaline substances.

The Morphology of the Parasites in the Insect and Avian Hosts.

The morphology of *Herpetomonas jaculum* and *H. culicis* in the insect and avian hosts shows little difference. Where the infection of the birds was of the chronic type, the non-flagellate, leishmaniform bodies were more numerous in organ smears, while in the acute cases the flagellate forms preponderated. We would point out that, while such was the case in these experiments of ours, we do not consider that any generalisation can yet be made therefrom. However, it may be noted that Monge (1914), dealing with the flagellate stages of *Leishmania tropica* in man in Peru, states that the presence of such flagellate stages may be an indication of increased virulence.

Herpetomonas jaculum, as found in our experimentally infected birds, showed non-flagellate and flagellate forms in various stages of growth and division. The non-flagellate parasites were from 4μ to $6{\cdot}6\mu$ long by 2μ to 5μ broad. They were oval, the nucleus being usually homogeneous but occasionally showing a karyosome, the blepharoplast occupying various positions in the body, as it does in the parasite as found in the insect.

Full grown flagellates were comparatively rare in the birds. They were morphologically like those in the insect host, but the maximum size, as in previous experiments, was not quite attained.

Herpetomonas culicis in the Culex, sparrow and martin had the same appearance. The non-flagellate forms were oval or pyriform, measuring 4μ to 6μ by 2μ to 4μ. Non-flagellates in various stages of division occurred. The flagellates were elongate, their body length varying from 11μ to 16μ, and their breadth from $1{\cdot}5\mu$ to $3{\cdot}6\mu$. The nucleus was usually granular, only occasionally karyosomatic. The blepharoplast was always conspicuous and varied from round to barlike. Multiplication of non-flagellate and flagellate forms by binary, longitudinal division occurred.

Natural Herpetomoniasis in Birds.

It is of much interest to note that in 1907 Drs Edm. and Et. Sergent published a short account of a herpetomonad which they found in the blood of a pigeon, when they were working on the relation of *Haemoproteus columbae* in pigeons to its second host, the fly, *Lynchia maura*. The body of the herpetomonad was straight and drawn out, measuring 17μ to 22μ. The flagellum measured 19μ to 35μ. The elongate nucleus was not as wide as the body. The blepharoplast was large, spherical and deeply staining.

The source of the herpetomonad is not known with certainty. We have heard from Dr Sergent that, so far as he was aware, his Lynchia were not infected with herpetomonads. It is possible that the bird may have had a latent herpetomoniasis contracted direct from insect hosts.

General Conclusions.

The general conclusions, resulting from the series of experiments presented to this Society, may now be shortly indicated. Under suitable conditions, insect flagellates can be introduced into vertebrate hosts and produce infection therein. In some cases, as in cold-blooded vertebrates, little obvious ill-effect results; in others, as in mammals and birds, disease is manifested. Similar infections are known to occur naturally in some cases, as in mice and pigeons.

The flagellates, such as herpetomonads, thus introduced, retain their powers of development on the same lines as when they were present in the insects. Leishmania has the same morphological cycle as Herpetomonas. The various species of Leishmania are probably insect herpetomonads, long since introduced into man and usually perpetuating the non-flagellate form, though capable of assuming the flagellate, herpetomonad facies in the internal organs of the vertebrate or in the invertebrate host.

Various vertebrates—fish, amphibia, reptiles, birds and mammals—may serve as reservoirs of leishmaniases. The virus may be very attenuated and so escape detection.

Leishmaniasis, which is a form of herpetomoniasis (leptomoniasis), is a flagellosis, as is also trypanosomiasis.

Summary.

1. Herpetomoniasis can be induced in birds, for example, canaries (*Serinus canarius*), sparrows (*Passer domesticus*), and martins (*Chelidon urbica*), by feeding them on material containing herpetomonads.

2. *Herpetomonas culicis* from *Culex pipiens* and *H. jaculum* from *Nepa cinerea* have infected birds when fed to them. The cycle of the flagellates in the avian hosts resembled morphologically that in the insects.

3. The disease induced may run an acute or a chronic course. In the acute cases in our birds, the flagellate form of the parasite was the more obvious at death. In the chronic cases, the non-flagellate forms were the more numerous.

4. Natural herpetomoniasis of a pigeon has been recorded by Drs Edm. and Et. Sergent in Algeria. This affords a parallel case with the natural and induced herpetomoniasis in mice previously recorded by us.

5. Members of all classes of vertebrates may be capable of acting as reservoirs of herpetomoniasis, and the virus may exist in a very attenuated condition and so be difficult of detection.

References.

Further references will be found at the ends of some of the papers listed.

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SERGENT, EDM. et ET. (1907). "Etudes sur les Hématozoaires d'Oiseaux." *Ann. Inst. Pasteur*, XXI, pp. 251–280, 2 Plates. (For the Herpetomonas in the pigeon, see footnote, p. 270 and Plate 7.)

The determination of the effective aperture of the stop of a photographic lens. By G. F. C. SEARLE, Sc.D., F.R.S., University Lecturer in Experimental Physics, Fellow of Peterhouse.

[*Read* 23 February 1914.]

§ 1. *Notation for stops.* The stops of a photographic lens are usually marked with the symbols $f/8$, $f/16$, &c. The symbol $f/8$ means that the effective diameter of the stop is one-eighth of the focal length of the lens system. This effective diameter is not the actual diameter of the hole in the diaphragm which is used to regulate the light entering the camera, but is, in the case of cameras adjusted for landscape photography, the diameter of that incident beam of rays parallel to the axis, which in its passage through the lens system exactly fills the opening in the actual diaphragm. The photographic speed of the lens, when used for landscape work, is proportional to the square of the diameter of the incident beam, and hence it is important to be able to test whether the numbers assigned to the various stops by the makers are correct. If, for instance, a maker marks a stop $f/8$ which is actually $f/10$, the effective area of the stop is only 64/100 of the area suggested by the maker's mark.

The "f" notation is not, however, the only system in use. The stops on some cameras are graduated in "U.S." numbers (Uniform System, not United States). The stops supplied with Kodak cameras are usually marked with the "U.S." numbers. On this system the stop denoted by $f/4$ by English makers is taken as the unit, and the numbers are so chosen that the "U.S." number of a stop is proportional to the exposure required. The correspondence between the systems is as follows:

"f" system	$f/4$	$f/8$	$f/16$	$f/32$	$f/64$
Uniform System	1	4	16	64	256

For intermediate stops, the relation may be expressed algebraically. If the "f" number is f/n, we have $f/n = f/(\frac{1}{4}n \times 4)$, and hence the corresponding "U.S." number is $(n/4)^2$. Thus, if f/n corresponds to the "U.S." number 8, we have $(n/4)^2 = 8$, or $n^2 = 128$, or $n = 11{\cdot}31$; this stop is often marked with the approximate number $f/11$.

The number n in the symbol f/n expresses the ratio of the focal length of the lens to the effective diameter of the stop in the case of landscape work. It is therefore convenient to call the reciprocal number $1/n$, as is done by some writers, the "aperture ratio" of the stop.

§ 2. *Stop in front of lens.* When the stop S (Fig. 1) is placed in front of the lens system, as was often the case in the older photographic lenses, its effective diameter for landscape work is

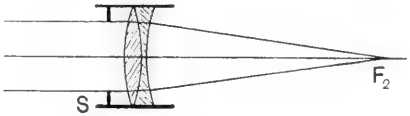

Fig. 1.

simply equal to the actual diameter of the stop itself. If the diameter of the stop is measured and is found to be a cm., and if the focal length of the lens is f cm., the aperture ratio of the stop is a/f and thus, if the "f" number is f/n,

$$1/n = a/f, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

and

$$n = f/a. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

§ 3. *Stop between components of lens system.* A stop does more than merely regulate the amount of light passing through the lens. The position of a stop of given aperture has an important influence upon the effects of the five "defects" of a lens, viz. upon the effects of Spherical aberration, Coma, Astigmatism, Curvature of image surfaces, Distortion; in the case of modern lenses consisting of two separated components, it is found that it is advantageous to place the stop *between* the components, as in Fig. 2. The effective diameter of the stop will now not be equal to its actual diameter. The effective diameter can be calculated from the actual diameter when the position of the stop relative to the two components and also the optical constants of the components are known, but it would be troublesome to obtain all these data. It is better to treat the lens system as a whole and to arrange the measurements so that there is no need to take the lens system to pieces.

Let S (Fig. 2) be the stop and let T_1 be the image of S formed by the front lens L. Then S is the image of T_1 by the same lens.

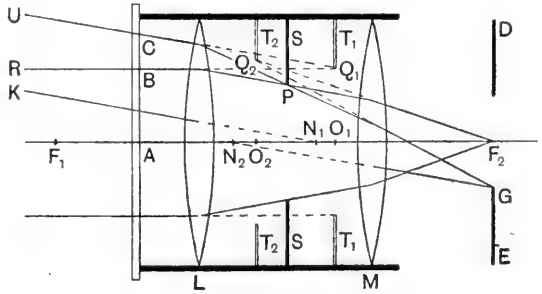

Fig. 2.

Hence, a ray RQ_1, which before incidence on the lens L is parallel to the axis AF_2 and is directed to a point Q_1 on the edge of T_1, will, after traversing L, pass through P, the corresponding point on the edge of the stop itself. Hence it is clear that the distance of the ray RQ_1 from the axis AF_2 is equal to the radius ($\frac{1}{2}a$) of the effective aperture.

If the front lens L is a converging lens, and if the stop S lies between L and the focal plane of L, the image T_1 will be further from L than S is and consequently the diameter of the aperture in the image will be greater than in the actual stop.

In Fig. 2, T_2 is the image of the stop S formed by the lens M. Each of the rays RB and UC in its path through the system passes actually, or at least in direction, through (1) Q_1, (2) P, (3) the edge Q_2 of T_2. It will be seen that Q_2 is the image of Q_1 by the complete lens system.

§ 4. *First method.* The effective diameter of the stop is easily measured by aid of a microscope mounted on a sliding carriage, such as that shown in Fig. 3. The lens system is *firmly* supported with its axis horizontal. The microscope is attached to the carriage so that its axis is at right angles to the direction of motion of the carriage, and the track on which the carriage slides is placed so

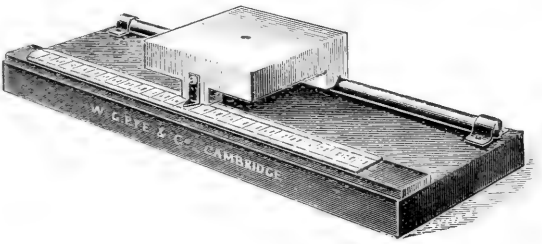

Fig. 3.

that the axis of the microscope is parallel to that of the lens system, the two axes being at the same height above the table. The microscope is then focussed through the lens L upon one end of the horizontal diameter of the stop, and the sliding carriage is adjusted so that the image of the edge of the stop is brought to coincidence with the cross-wire of the microscope or with a selected dividing line of the micrometer scale. Some adjustment of the microscope in the direction of its own axis will be required; the adjustment is complete when there is no parallax between the image and the cross-wire. The carriage is then moved along its track so as to bring the image of the other end of the diameter to the cross-wire or to the same micrometer division as before. The distance through which the microscope has been moved is equal to a, the effective diameter of the stop. After each pair of

readings the track may be slightly moved so that the next pair of readings is quite independent of those already taken. If the microscope has a micrometer scale in the eyepiece, a different dividing line may be used for each pair of readings for the two ends of the diameter.

The focal length, f, of the system is then found by any method—the goniometer method or the revolving table method is convenient*—and the value of $n = f/a$ is calculated.

§ 5. *Second method.* If a luminous point were placed at the focus F_2 (Fig. 2), the rays from it, after passing through the system, would form a beam of rays parallel to the axis AF_2. If a glass scale were placed against the mount of the lens, as ABC in Fig. 2, the diameter of the beam of light could be at once read off, provided that the face of the scale which rests against the mount has a suitable matt surface, *i.e.* a surface which will so scatter any light which falls upon it that it is easy to distinguish the parts which are illuminated from the parts which are not illuminated. The *divided* face of the glass scale should rest against the mount. The diameter of the bright patch on the scale is equal to the effective diameter of the stop†.

The practical method of obtaining a beam of rays from a small area in the focal plane is to allow light from a flame to pass through a small opening in a thin metal plate placed in that plane. The source of light must, of course, be near enough to the plate to ensure that the stop is filled with light; it need not be so near as to *burn* the support of the plate. But, since the rays do not now proceed from a *point*, the emergent beam will not be made up only of rays which are parallel to the axis and consequently the diameter of the bright patch on the scale ABC will no longer be equal to the effective diameter of the stop. The necessary correction is, however, easily found.

Let G be a point, in the plane of the diagram, on the edge of the circular opening in the plate in the focal plane, let N_2 be the nodal point of the lens system corresponding to F_2, and let N_1 be the other nodal point. Then, if a ray from G is directed towards N_2 before it strikes the lens M, it will, on emergence from the lens L, proceed in the direction N_1K as if it came from N_1, the two directions GN_2 and N_1K being parallel, by virtue of the property

* For a description of these methods see G. F. C. Searle, "Demonstration of Laboratory Apparatus and Experiments," *Proceedings of the Optical Convention*, 1912, p. 161.

† A matt surface may be obtained by aid of paraffin wax. The scale is *gently* heated over a flame until it is warm enough to melt the wax. A thin layer of wax may be obtained by wiping off most of the melted wax by a piece of paper with a straight edge. With a little practice, a suitable coating of wax is easily obtained. Mr R. Whiddington found that a good matt surface may be obtained by dabbing a piece of putty against the glass.

of the nodal points. Since G is in the focal plane, *all* the rays due to G which emerge from the lens L are parallel to N_1K. The extreme ray is the one which, in its passage through the system, passes through the point P on the edge of the stop, P and G both lying in the plane of the diagram. But Q_1 is the image of P formed by the lens L, and thus the extreme ray, on emerging from L, must be directed away from the point Q_1. Hence, if Q_1CU be drawn parallel to GN_2, it will give the direction of the extreme emergent ray. If C is the point in which it strikes the matt surface of the glass scale ABC, AC will be the radius of the bright patch on the scale*.

Since CQ_1 is parallel to N_2G, the triangles CBQ_1, GF_2N_2 are similar, and hence $CB/BQ_1 = GF_2/F_2N_2$.

If d is the diameter of the opening in the focal plane, then $F_2G = \frac{1}{2}d$. If $BC = \frac{1}{2}b$ and $BQ_1 = x$, we have $b/x = d/f$ or

$$b = xd/f. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Hence, if c is the diameter of the bright circular patch on the matt surface of the scale,

$$c = a + b = a + xd/f, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

and therefore

$$a = c - xd/f. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

The distance $BQ_1 = x$ is the distance between the matt surface of the scale and the plane of the image of the stop formed by the lens L. A plate of transparent glass of good quality is fixed against the mount of the lens in place of the glass scale (the latter may be used if the surface is cleaned) and a little lycopodium is placed on the face nearest to L. A microscope on a sliding carriage is then arranged so that its axis and the direction of motion of the carriage are parallel to the axis of the lens system, and the microscope is focussed through the glass plate ABC first on the edge of the stop and then on the lycopodium, the adjustment being effected by sliding the carriage along its track. The distance through which the microscope is moved is equal to BQ_1 or x. After each pair of observations the track may be slightly moved so as to obtain independent readings for the next pair.

* It has been assumed that the aperture in the metal plate S (Fig. 2) really acts as a stop for rays which converge to F_2. In other words, it is assumed that the effective diameter of the incident beam is limited by S and not by the mountings of the lenses. If the aperture in S is so large that the ray which passes through P in its course to F_2 passes close to the lens mountings, it may happen, as Mr T. Smith has pointed out to me, that, unless F_2G is small, the ray from G which is directed to Q_2 is caught by the lens mountings and does not penetrate the system. In that case the edge of the luminous patch is not at C but at some point nearer to A, and thus the diameter of the patch depends not on S but on the diameter of one of the lens mountings. If such a system were used for landscape photography, there would be a great falling off of illumination towards the edges of the plate. This defect is avoided in good photographic lenses by keeping the stop of greatest aperture sufficiently small.

The diameter d of the hole in the plate may be measured by a glass scale placed with its divided face in contact with the plate. If desired, greater accuracy could be obtained by using a travelling microscope.

§ 6. *Third method.* The determination of the distance BQ_1 may be avoided by using *two* plates with apertures of diameters d_1 and d_2. Let c_1 and c_2 be the diameters of the corresponding bright patches on the matt surface of the glass scale. Since, by (5), $c - a$ is proportional to d, we have

$$(c_1 - a)(c_2 - a) = d_1/d_2.$$

Thus $$a(d_1 - d_2) = c_2 d_1 - c_1 d_2 = c_1(d_1 - d_2) - d_1(c_1 - c_2),$$

and hence
$$a = c_1 - \frac{d_1(c_1 - c_2)}{d_1 - d_2}. \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

§ 7. *A system for laboratory work.* A projection lens system such as is used in optical lanterns is convenient for the experiment. Such lenses are moderate in price (10 - upwards) and are reasonably well corrected. The cylindrical tube which forms the mount of the lens is, on account of its form, easily attached to a base. The stop may be inserted in the tube between the two components of the lens system. Fig. 4 shows the arrangement which has been found convenient at the Cavendish Laboratory. The projection lens AB, of about 8 inches focal length, is mounted at one end of a base-board. At the other end of the board is fixed a wooden upright C with a circular opening. The metal plate D rests against that face of the upright which is furthest from the lens and is held in position by a rubber band or a spring. The lens is adjusted on

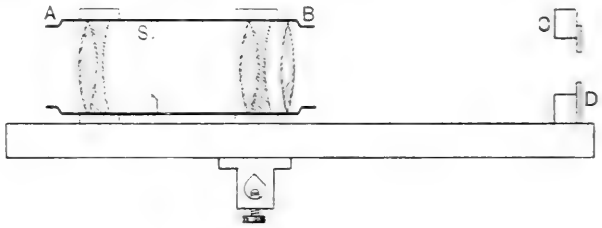

Fig. 4.

the board so that the focal plane of the lens accurately coincides with the face of C against which D rests. The stop S should have a bevelled edge. A screw clamp fixed to the underside of the base-board allows the arrangement to be attached to an iron rod carried by a heavy base.

If the focal length is measured by the goniometer method, a

glass scale is substituted for the plate D, its divided face being in contact with the upright.

Though this arrangement is more convenient for daily laboratory work than an actual camera, students who possess cameras find the work more interesting if they investigate the stops on their own cameras.

A projection lens so mounted is a very useful piece of apparatus. If a cross-wire is fitted to the metal plate D, the arrangement acts as a good collimator. The spherical aberration will be small if the lens system is mounted on the board so that the end which is intended to face the lantern slide, when the lens is used for projection, faces the plate D. If the lens is mounted the wrong way round, its performance will be much less satisfactory.

§ 8. *Practical example.* The following results were obtained with an optical lantern projection lens system.

First method.

The readings of the sliding carriage on the track, when the microscope was focussed on the two sides of the stop in turn, were as follows:

Right edge of image of stop,	20·97,	20·91,	21·00 cm.
Left edge of image of stop,	17·78,	17·72,	17·82 cm.
Differences,	3·19,	3·19,	3·18 cm.

Hence (§ 4), mean value of effective diameter of stop $= a = 3{\cdot}19$ cm.

Second method.

Diameter of aperture in metal plate $= d_1 = 0{\cdot}80$ cm.

The readings on the glass scale of the edges of the bright patch were as follows:

Right edge of bright patch,	9·28,	9·30,	9·35 cm.
Left edge of bright patch,	5·97,	6·00,	6·04 cm.
Differences,	3·31,	3·30,	3·31 cm.

Mean value of diameter of patch $= c_1 = 3{\cdot}31$ cm.

It will be seen that c_1 is considerably larger than a as found by the first method.

For the distance BQ_1 of the image of the stop from the plane of the glass plate the readings were:

Microscope focussed on B,	11·96,	11·92,	11·93 cm.
Microscope focussed on Q_1,	8·52,	8·53,	8·50 cm.
Differences,	3·44,	3·39,	3·43 cm.

Mean value of $BQ_1 = x = 3{\cdot}42$ cm.

The focal length of the system was found by the revolving table method. The two readings on the scale were 46·20 and 3·02 cm.

Hence $$f = \tfrac{1}{2}(46{\cdot}20 - 3{\cdot}02) = 21{\cdot}59 \text{ cm.}$$

Thus, by (5),

$$a = c_1 - \frac{xd_1}{f} = 3{\cdot}31 - \frac{3{\cdot}42 \times 0{\cdot}80}{21{\cdot}59} = 3{\cdot}31 - 0{\cdot}13 = 3{\cdot}18 \text{ cm.}$$

Third method.

Here two apertures were used. The diameter of the first was $d_1 = 0{\cdot}80$ and that of the second was $d_2 = 0{\cdot}42$ cm. The diameter of the bright patch due to the first was $c_1 = 3{\cdot}31$, as found above. The readings for the bright patch for the second aperture were as follows:

Right edge of bright patch,	7·90, 8·05, 8·22 cm.
Left edge of bright patch,	4·64, 4·80, 4·97 cm.
Differences,	3·26, 3·25, 3·25 cm.

Mean value of diameter of patch $= c_2 = 3{\cdot}25$ cm.
Hence, by (6), we find for the effective diameter of the stop,

$$a = c_1 - \frac{d_1(c_1 - c_2)}{d_1 - d_2} = 3{\cdot}31 - \frac{0{\cdot}80 \times 0{\cdot}06}{0{\cdot}38} = 3{\cdot}31 - 0{\cdot}13 = 3{\cdot}18 \text{ cm.}$$

The mean of the three values gives $a = 3{\cdot}18$ cm. Hence

$$n = f/a = 21{\cdot}59/3{\cdot}18 = 6{\cdot}79.$$

The marking of the stop on the "f" system is therefore $f/6{\cdot}79$, and the aperture ratio is $1/6{\cdot}79$ or $0{\cdot}147$.

§ 9. *Effect of distance of object on effective aperture.* The foregoing discussions refer to the case in which the light which falls on the photographic plate comes from an object at an infinite distance so that the plate is in the focal plane of the lens system. When, however, the object is not at infinity, as when the camera is used in copying, the plate will not be in the focal plane F_2G and a fresh investigation becomes necessary. It appears, however, that the speed of the lens can be calculated for any position of the object, when, in addition to the data obtained in the second method (§ 5), we know the position relative to the lens system of the other principal focus F_1.

The speed of the lens will be proportional to the light which reaches unit area of the image when the object is a surface of standard brightness. Let an infinitesimal element dS of this surface be placed at O (Fig. 5), the centre of a geometrical hemisphere of unit radius, the plane of the element coinciding

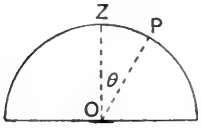

Fig. 5.

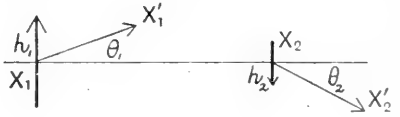

Fig. 6.

with the base of the hemisphere. Let Ids be the light which passes, per unit area, through the surface of the hemisphere in the neighbourhood of the pole Z, where ZO is normal to dS. Then I is called the intrinsic brightness of the surface.

The light which passes, per unit area, through the surface of

the hemisphere in the neighbourhood of P, where $POZ = \theta$, is $IdS \cdot \cos\theta$. Hence the light which passes through the spherical cap, which contains Z and is bounded by the line of latitude passing through P, is

$$IdS \int_0^\theta 2\pi \sin\theta d\theta \cdot \cos\theta = \pi I \sin^2\theta \cdot dS.$$

Suppose, now, that the object is a disk of radius h_1 placed with its centre at X_1 on the axis of a lens system, the normal to the disk coinciding with the axis, and that its image is a disk of radius h_2 with its centre at X_2. The arrows in Fig. 6 serve to show that, in the case figured, the image is inverted. Let the ray X_1X_1', after passing through the system, become the ray X_2X_2' and let these rays make angles θ_1 and θ_2 with the axis; in the figure h_2 and θ_2 are negative. We shall suppose that *all* the rays from the point X_1 which pass through the system meet again in the point X_2, so that the system is free from spherical aberration with respect to the two points X_1, X_2. Then, if *every* ray which leaves any point on the edge of the disk at X_1 passes through the corresponding point on the edge of the image disk after traversing the system, so that there is no coma, then h_1, h_2, θ_1, θ_2 will satisfy the "sine condition"

$$\mu_1 h_1 \sin\theta_1 = \mu_2 h_2 \sin\theta_2, \quad \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

where μ_1 and μ_2 are the refractive indices of the media at X_1 and X_2. When, as in our case, there is air at both ends of the system, $\mu_1 = \mu_2 = 1$ and then

$$h_1 \sin\theta_1 = h_2 \sin\theta_2. \quad \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

It is assumed that the radii h_1 and h_2 are infinitesimal.

If the lenses absorb no light, all the light, which leaves the object disk at X_1 and traverses the system, reaches the image disk at X_2. If X_1X_1' be the extreme ray from X_1 which passes through the system, the amount of light which reaches the image is $\pi I \sin^2\theta_1 \cdot \pi h_1^2$; by (8), this is the same as $\pi I \sin^2\theta_2 \cdot \pi h_2^2$. But the latter is exactly the light which would pass out from the image disk within the angular limits defined by θ_2, if it were an actual disk with a surface of intrinsic brightness I. The image may therefore be described as having the same intrinsic brightness as the object. The equality $\pi I \sin^2\theta_1 \cdot \pi h_1^2 = \pi I \sin^2\theta_2 \cdot \pi h_2^2$ expresses the important reciprocal result that, if the two disks are of equal intrinsic brightness, the light which one receives from the other is the same for both.

If the light which reaches unit area of the image be denoted by L, we have

$$L = \pi I \sin^2\theta_2, \quad \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

and thus the speed of the lens will be proportional to $\sin^2\theta_2$.

The system cannot be free from spherical aberration and from coma for all pairs of conjugate points, and it thus becomes necessary to accept a lower standard and to work on that plane of approximation where the angles are treated as being so small that they may be used instead of their sines or tangents. We then have

$$L = \pi I \theta_2^2. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(10)$$

We can now apply these results to determine the effect of the position of the object upon the speed of the lens.

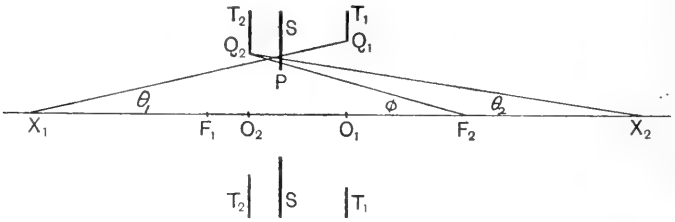

Fig. 7.

In Fig. 7, let S be the actual stop, T_1 its image formed by the front lens L (Fig. 2) and T_2 its image formed by the back lens M; the lenses themselves are not shown in Fig. 7. Let $O_1Q_1 = q_1$ and $O_2Q_2 = q_2$ be the radii of the two images of the opening of the stop. Then q_1 is identical with $\frac{1}{2}a$, where a is the quantity used in §§ 2, 4. If X_1 is a point on the axis and X_2 is its image, one of the extreme rays leaving X_1 and reaching X_2 is the incident ray X_1Q_1 which gives rise to the emergent ray Q_2X_2. Let these make angles θ_1, θ_2 with the axis, where $\theta_1 = q_1/O_1X_1$ and $\theta_2 = q_2/O_2X_2$. Let ϕ be the angle subtended by Q_2O_2 at the focus F_2, where the plate is placed when a distant scene is to be photographed. Then $\phi = q_2/O_2F_2$. We see, at once, that, when X_2 is beyond F_2, as is the case when a "real" object is to be photographed, the angle θ_2 is less than the angle ϕ and that, in consequence, the speed of the lens is less when the object is at a finite distance than when it is at an infinite distance from the lens, for, by (10), the speeds are proportional to θ_2^2 and ϕ^2.

If L_∞ be the light received per unit area by the image when the object surface of standard intrinsic brightness I is at an infinite distance, we have

$$L = \pi I \theta_2^2 = \pi I q_2^2 / O_2X_2^2,$$

$$L_\infty = \pi I \phi^2 = \pi I q_2^2 / O_2F_2^2,$$

and thus

$$\frac{L_\infty}{L} = \left(\frac{O_2X_2}{O_2F_2}\right)^2 = \left(\frac{O_2F_2 + F_2X_2}{O_2F_2}\right)^2 = \left(1 + \frac{F_2X_2}{O_2F_2}\right)^2. \quad \ldots(11)$$

Since X_2 is the image of X_1 and O_2 is the image of O_1 formed in each case by the *complete* lens system, we have, by Newton's formula,

$$F_2X_2 \cdot F_1X_1 = f^2, \quad O_2F_2 \cdot O_1F_1 = f^2,$$

and hence
$$\frac{L_\infty}{L} = \left(1 + \frac{O_1F_1}{F_1X_1}\right)^2 = \left(\frac{F_1X_1 + O_1F_1}{F_1X_1}\right)^2, \quad \ldots\ldots\ldots(12)$$

and thus
$$\frac{L}{L_\infty} = \left(\frac{F_1X_1}{F_1X_1 + O_1F_1}\right)^2. \quad \ldots\ldots\ldots(13)$$

Hence the intensity of illumination of the image diminishes as the object approaches the focus F_1 from infinity. If we measure the distance between O_1 and F_1 we can find the value of L/L_∞ for any given value of F_1X_1. In the *second method* (§ 5) the distance, x, of O_1 or Q_1 from the plane of the end of the lens mount was determined. The focus F_1 can be identified by placing a pin so that its tip coincides with the image of a very distant object, or by making the tip coincide with its own image when a plane mirror is placed on the other side of the lens system. *After* the pin has been placed at F_1, a plate of glass or metal of known thickness is placed against the mount and the distance of the pin from this plate is measured. In this way the distance of F_1 from the plane of the end of the mount is found. Adding this to the distance x, we obtain O_1F_1.

The effect of the finite distance of the object in diminishing the illumination of the image may be met either by increasing the time of exposure or by using a larger stop.

If, for a given stop, the time of exposure, for an object surface of standard intrinsic brightness, is t_∞ when the object is at infinity, and is t when the object is at a finite distance, then

$$\frac{t}{t_\infty} = \frac{L_\infty}{L} = \left(\frac{F_1X_1 + O_1F_1}{F_1X_1}\right)^2. \quad \ldots\ldots\ldots(14)$$

If the camera is used for copying and the linear dimensions of the copy are to be m times those of the original, we must have $F_1X_1 = f/m$. We then obtain, if $O_1F_1 = \beta f$,

$$\frac{t}{t_\infty} = \left(\frac{f + mO_1F_1}{f}\right)^2 = (1 + m\beta)^2. \quad \ldots\ldots\ldots(15)$$

If the lenses are not very thick and are not widely separated, O_1F_1 will not differ much from f, and thus β will be nearly unity. In this case we obtain approximately

$$t = (1 + m)^2 t_\infty.$$

If we wish to keep the exposure unaltered, we must increase the stop aperture when the object is at a finite distance. If the stop marked f/n_∞ is suitable when the standard surface is at an

infinite distance, and if the stop marked f/n is to be used when the surface is at a finite distance, then $n^2t = n_\infty{}^2 t_\infty$, where t/t_∞ is given by (14) or (15). Hence

$$n = n_\infty \left(\frac{t_\infty}{t}\right)^{\frac{1}{2}} = n_\infty \cdot \frac{F_1X_1}{F_1X_1 + O_1F_1} = n_\infty \cdot \frac{1}{1+m\beta}. \quad \ldots(16)$$

§ 10. *Practical example.* The same system was used as in § 8.

The distance from F_1 to the plane of the end of the lens mount was 16·16 cm. The distance x between that plane and O_1 was found in § 8 to be 3·42 cm. Hence $O_1F_1 = 16{\cdot}16 + 3{\cdot}42 = 19{\cdot}58$ cm.

The focal length f was found in § 8 to be 21·59 cm.

Hence $\beta = O_1F_1/f = 19{\cdot}58/21{\cdot}59 = 0{\cdot}907$.

The exposure for a given stop can then be calculated by (15) and the stop for a given exposure can be found by (16).

PROCEEDINGS

OF THE

Cambridge Philosophical Society.

A preliminary account of the structure of the mouth-parts in the Body-louse.

By LAUNCELOT HARRISON, B.Sc., *Exhibition of* 1851 *Research Scholar of the University of Sydney, Emmanuel College, Cambridge.* (*From the Quick Laboratory, University of Cambridge.*) (Communicated by Prof. G. H. F. NUTTALL, F.R.S.)

[Read 21 *February* 1916.]

[Plate VII.]

THE Body-louse is usually referred to as *Pediculus vestimenti* Nitzsch, occasionally as *P. corporis* Degeer. These names were given respectively in 1818 and 1778. The correct name is that given by Linné in the 10th edition of the *Systema Naturae*, 1758, namely *Pediculus humanus*.

The mouth-parts of this insect have a unique interest in zoological literature, owing to the controversy which raged for many decades as to whether the louse bit or sucked. Inability to settle this question was held up as a stock reproach to biologists of the first half of the nineteenth century. Schiödte (1866) finally cleared up this point, and his account of the method of feeding the louse still stands as the best written. But although he definitely established the suctorial nature of the apparatus, its structure has remained something of a mystery, and its homologies have equally remained dubious. Within the last dozen years a somewhat bitter controversy has appeared in the pages of the *Zoologischer Anzeiger*, between Professor Cholodkowsky and Dr Enderlein, in which diametrically opposed views of both structure and homology have been put forward by the protagonists. Pawlowsky (1906), a pupil of Cholodkowsky, set himself to clear

the matter up finally. His account, though an advance on any previously published, fails in several important respects. Patton and Cragg (1913) have also described the mouth-parts of *Pediculus*, but, although they have given a more correct account of the structure of the piercing apparatus, their description of the buccal cavity and pharynx, and more especially of the musculature, falls behind that of Pawlowsky. The paper of the latter gives an excellent summary of the history of the controversy, from Swammerdam to the date of his own paper.

Material and Methods.

This paper sets out part of the results of an enquiry into the structure and bionomics of *Pediculus* which was undertaken by the staff of the Quick Laboratory at the instance of the Medical Department of the Local Government Board. As it is largely concerned with an interpretation that is of academic rather than practical interest, I take this opportunity of publishing my interpretation somewhat fully, so that I need not encumber an official report with more than a summary of it. Material has been furnished by various medical officers in the service of the Board, and was also obtained at the 1st Eastern General Hospital, from the clothing of wounded soldiers. This material was fixed in Carnoy's fluid. For the preparation of satisfactory sections I have to thank Dr Keilin, of the Quick Laboratory, who placed his very considerable technical experience entirely at my service, and developed a technique suitable for dealing with this difficult material. No reagent was used to soften the chitin, as experience with Mallophaga had shown me that the use of strong reagents, such as Eau de Labbaraque, is not altogether satisfactory. The difficulty of securing satisfactory infiltration was finally overcome by embedding *in vacuo*, and sections were cut at 6μ, and stained in Mann's stain. In addition to longitudinal and transverse sections of the head, dissections, with and without the use of potash, were made, both of *Pediculus* and of the larger Anopluran species, *Haematopinus tuberculatus* and *H. suis*. Freshly killed insects cleared in glycerine also proved useful; while, for the structure of the chitinous parts, moulted skins of the larvae were supremely useful, as a beautiful cast of these parts can thus be obtained.

General Relations.

The stomatodaeum of *Pediculus* is divisible into four distinct portions, a *buccal cavity*, *pumping-pharynx*, *pharynx* proper, and *oesophagus*. The *buccal cavity* is an approximately cylindrical tube, leading horizontally from the terminal *mouth* opening to the *pumping-pharynx*. It is ·13 mm. in length, and is wider posteriorly

than anteriorly. Upon the floor of this cavity, near its hinder end, opens a long invagination, the *piercer-sheath*, which extends beneath the alimentary canal back almost to the occiput, and contains the *piercing apparatus*. The *pumping-pharynx* is, in its dilated condition, ellipsoid, its long axis corresponding to that of the head, its posterior end higher than the anterior, its length ·12 mm. In the resting condition it gives much the same section in both longitudinal and transverse sections, the floor and roof, with reference to the lumen, being concave and convex respectively. The *pharynx*, which has a length of ·1 mm., is situated dorsally between the anterior cornua of the brain, and is X-shaped in cross section owing to the action of powerful sphincters. It descends somewhat posteriorly, and passes into the *oesophagus* a narrow tube which passes downwards between the brain and sub-oesophageal ganglion, and runs backwards to join the mid-gut towards the posterior margin of the thorax. This gullet has an approximate length of ·5 mm.

Within the *piercer-sheath* lie four structures, which are directly continuous with six chitinous tendons, into which six muscles rising from the posterior wall of the head-capsule are inserted. The tendons of the dorsal muscles are continued into the *dorsal-piercer*, which throughout the greater part of its length has the form in section of two brackets lying side by side thus ⌣⌣, with their contiguous edges fused. The tendons of the lateral pair of muscles concresce as the *ventral-piercer*, the precise structure of which will be discussed later. The tendons of the ventral muscles expand into a chitinous plate, separable into anterior and posterior portions, which lies embedded in the floor of the sheath. These plates may represent the *mentum* and *submentum* of the *labium*. The fourth structure is a chitinous duct which is first attached by a strand of tissue to the ventral side of the dorsal-piercer, then becomes free, and lies in a groove on the dorsal surface of the ventral-piercer, and finally enters the buccal cavity equidistant between the two. This duct originates in paired strands of tissue arising from the ventral tendons, and passing into relation with the dorsal-piercer, and is, I believe, the *salivary duct*, although I have not been able to trace a definite connection with certainty, the structures being excessively minute.

The structures so far indicated have been described with more or less success by previous workers. One important part of the apparatus has, however, hitherto altogether escaped notice. This part, which I call the *buccal tube*, is a tube lying free within the buccal cavity, composed of two lateral apposable half-tubes which arise ventro-laterally from the floor of the fore-gut at the point of junction of the buccal cavity and pumping-pharynx. Into the

lumen of the tube thus formed enter the dorsal and ventral piercers, and the salivary duct.

Some of the interpretations which have been placed upon the structures thus briefly indicated will appear in the more detailed descriptions that follow. I treat the structure and functions together for each of the several parts which have been enumerated; and follow this by a recapitulary account of the whole process of feeding, with the combination of functions involved.

The mouth and buccal cavity.

In the position of rest, the mouth is an oval aperture, with longer diameter transverse, absolutely terminal. It is surrounded dorsally and laterally by loosely folded integument. It leads into the approximately cylindrical buccal cavity, on the dorsal wall of which, just inside the mouth, are two closely apposed chitinous plates, bearing a number of recurved denticles. At their proximal ends these *dental plates* are loosely hinged to the *buccal plate*, a

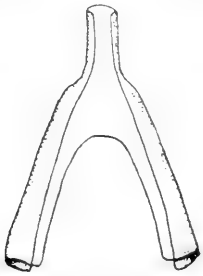

Fig. 1. Isolated buccal plate of larva, dorsal.

thick chitinous structure which lines the roof and lateral walls of the cavity, and which is thickest dorsally, gradually thinning out as it approaches the floor. Fig. 1 shows a dorsal view of this structure as it appears in a moulted cast; and it is shown in

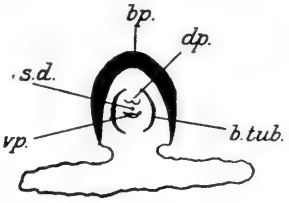

Fig. 2. Transverse section through buccal cavity just behind mouth. Diagrammatic.

section in Figs. 2 and 3 (*bp.*). At the posterior end of the buccal cavity this plate divides about the pumping-pharynx, in

the walls of which it continues backwards as two divergent *cornua* (Figs. 4 and 5, *rcbp.*, *lcbp.*). These cornua are the *fulturae* of Enderlein, the plate itself being his '*untere Lamelle des Pharynx*' (1904, p. 125, Fig. 2). To the ends of the cornua are attached three pairs of muscles, which have been described by Pawlowsky. The *dorsal protractors* (*lateral protractors* of Pawlowsky) of the right and left side (Figs. 4, 5, *rdp.*, *ldp.*, Plate VII, Fig. 1, *rdp.*) run forward from the corresponding cornua to be inserted into the

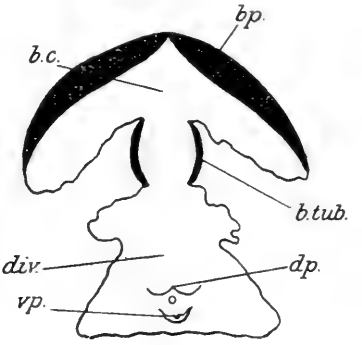

Fig. 3. Transverse section through buccal cavity at its function with the piercer diverticulum. Diagrammatic.

dorsal wall of the head above the mouth; the *ventral protractors* (same Figs., *rvp.*, *lvp.*) run from the underside of the cornua to the ventral wall of the head below the mouth; while the third pair, the right and left *retractors of the buccal cavity* (Plate VII,

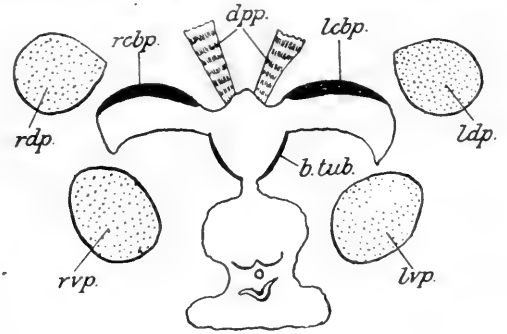

Fig. 4. Transverse section just posterior to Fig. 3. Diagrammatic.

Fig. 1, *rrbc.*) run from the cornua backwards, upwards, and outwards, to be inserted into the dorsal wall of the head above the brain.

By contraction of the four protractor muscles, the rigid buccal plate is impelled forwards, and everts the integumentary folds already mentioned as lying dorsally and laterally about the mouth opening, so that a small hood-shaped projection, open ventrally, is formed in front of the head. This is the *haustellum* of the older writers. In the process of eversion, the teeth on the dental plates are revolved outwards, and not only effect a superficial fixation of the parasite upon the skin, but also, owing to their diverging paths, tightly stretch the skin. It should be noted that, since this haustellum is open on the ventral side, it cannot form a closed chamber when in contact with the skin of the host, as is generally assumed. On relaxation of the protractors, and contraction of the retractors, the buccal plate is drawn backwards, and the haustellum disappears.

The ventral portion of the buccal cavity is somewhat expanded laterally throughout its length, and the cavity is a little wider posteriorly than anteriorly.

The pumping-pharynx.

The pumping-pharynx lies between the cornua of the buccal plate, projecting somewhat behind their hinder ends. The cornua are in this region bent downwards (Fig. 5) so as to partly surround, above and at the sides, two lateral cavities which are part of the

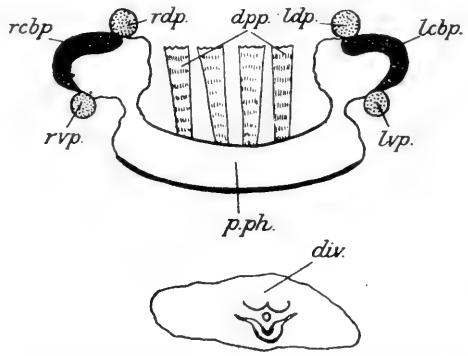

Fig. 5. Transverse section through pumping-pharynx about the middle of its length. Diagrammatic.

pumping-pharynx, but which are almost completely cut off from it in the resting condition. In this condition the pumping-pharynx resembles an early gastrula, or a collapsed rubber ball, of which the upper half has been pushed down into the lower. A large number of dilatator muscles (Figs. 4, 5, Plate VII, Fig. 1, *dpp.*) are inserted into this depressed roof, which run to the dorsal

and dorso-lateral regions of the head wall, and which, upon contraction, raise the depressed roof, so that the chamber assumes an ellipsoid shape. In this condition the lateral cavities supported by the cornua are not nearly so conspicuous. In specimens which have been treated in potash, and then passed into glacial acetic acid, the chamber is invariably blown out into this ellipsoid shape. Posteriorly the lumen narrows, and ascends towards the pharynx. At the narrowest part a band of hair-like chitinous processes of the cuticle projects into lumen all round. These processes are referred to by Pawlowsky, who suggests that they may assist in closing the lumen, but I cannot see how they can serve such a purpose. They show no indication of any sensory function, and probably act as a mechanical straining apparatus to prevent any foreign particle from entering the pharynx. They are indicated in Plate VII, Fig. 1, *ps.*

A valvular apparatus exists at the anterior end of the pumping-pharynx, which will be described in connection with the buccal tube. The function of this part of the stomatodaeum is very obvious. By the action of the dilatator muscles in raising the depressed roof, a negative pressure results, and blood is drawn into the cavity. By the closure of the anterior-valve, and the relaxation of the dilatators in order from before backwards, the blood is forced backwards into the pharynx. That the dilatators do relax in this way, and not all together, seems certain from a study of the feeding insect under a low power binocular, as the blood can be plainly seen to come and go in a manner reminiscent of the peristaltic process in the dorsal vessel of an annelid.

Certain of the muscles inserted into the roof would seem to be at least partially concerned in the protraction of the pumping-pharynx, as they run forwards, as well as upwards, to be inserted into the ring of thickened chitin which almost completely surrounds the head at the level of the opening of the piercer-sheath into the buccal cavity.

The pharynx and oesophagus.

An ascending tube, slightly bent upon itself, and very short, connects the pumping-pharynx with the pharynx. I cannot follow Patton and Cragg's account of this portion of the alimentary canal. They seem to me to have confused a short dorsal diverticulum at the anterior end of the pharynx, which for three or four successive sections appears as a separate cavity, and then cuts out, as a fold in the connecting tube. At its anterior end the pharynx shows a deep, fairly narrow, lumen in transverse section, which almost immediately becomes reduced to a smaller, X-shaped, lumen, owing to the presence of a powerful *anterior sphincter* (*as.* in Plate VII, Fig. 1). Behind the anterior sphincter three pairs of

dilatator muscles are inserted into the pharyngeal walls, the *right* and *left dorsal, lateral,* and *ventral dilatators* of the pharynx (*rdd., rvd.*). The dorsal pair are inserted dorso-laterally into the pharyngeal wall, and pass upwards, and slightly outwards and forwards, to attach to the wall of the head close to the mid-line. The lateral pair run outwards, and slightly upwards and backwards, to be inserted into the head-wall immediately above the eyes. The ventral pair run downwards, outwards, and slightly backwards, to be inserted into the head-wall midway between the eye and the mid-ventral line. This muscle is thus considerably displaced in Fig. 1 of Plate VII, which has been slightly schematised for the sake of clearness. Immediately behind these dilatator muscles a *posterior sphincter* (*psp.*) occurs; and behind this a pair of *posterior dorsal dilatators* (*rpdd.*), which have the same relations as the anterior pair. Pawlowsky attaches the ventral dilatators behind the posterior sphincter, but they are inserted between the sphincters in the same plane as the dorsal and lateral dilatators.

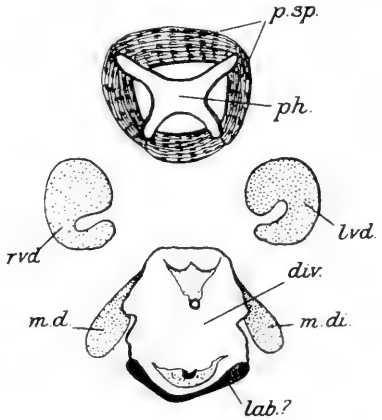

Fig. 6. Transverse section through pharynx and diverticulum at level of eyes. Diagrammatic.

The function of the pharynx would appear from its musculature to be twofold. Contraction of the sphincters closes the cavity, and allows of the production of a partial vacuum in the pumping-pharynx. Simultaneous dilatation of the pharynx and relaxation of the roof of the pumping-pharynx causes the blood to pass from the latter chamber to the former. The next contraction of the sphincters, and it seems probable here also that the anterior sphincter contracts first, forces the blood into the gullet.

The oesophagus is a narrow tube passing downwards from the posterior end of the pharynx between the brain and sub-oesophageal

ganglion. It then bends upwards and enters the thorax between the main tracheal trunks, and opens into the ventriculus towards the hinder margin of the thorax. Pawlowsky states (1906, p. 202) that the wall of the oesophagus contains no muscular elements, but this is not the case. In sections they are not discernible, but in dissections the appearance of the wall leaves no doubt that both longitudinal and transverse fibres are present.

I have not been able to make out any duct entering any part of the stomatodaeum so far described.

The buccal tube.

I first observed this structure in glycerine cleared larvae, in which it appeared as a delicate hyaline tube projecting into the buccal cavity, and seemingly in direct continuity with the pumping-pharynx, separated only by what looked like a valvular apparatus folded back into the cavity of the latter. The wall appeared to consist of delicate white chitin, transversely striated. At this time I had no sections, and as the structure had not been described in any previous account, I was quite at a loss to account for it, especially as it was not comparable with any structure with which I was familiar in the comparative anatomy of the insect mouth.

In section it is shown to be formed of two apposable half-tubes (Figs. 2, 3, *b.tub.*), and not of a single tube. These half-tubes take origin from the floor of the buccal cavity at its junction with the pumping-pharynx, and immediately behind the opening of the piercer-sheath. Pads of tissue of a peculiar kind underlie their bases, which appear to contain muscular elements, but the precise histological nature of which I have not been able to determine. After lying in the wall of the buccal cavity for a short distance, the half-tubes become free, and run forward as chitinous structures, forming a tube with approximately half the diameter of the surrounding cavity. The piercing apparatus enters into this tube through the ventral fissure, and runs forward within it. The tube ends abruptly beneath the end of the buccal plate, the piercer in the retracted condition projecting a little beyond its anterior end. How Pawlowsky missed this structure I cannot imagine, since he has figured a section (1906, p. 201, Fig. 8) passing through the base of it, which he letters as the *mundhohle*, though the large buccal cavity is shown lying dorsal to it.

The exact relations of what I have referred to above as a valvular apparatus cannot be satisfactorily determined from sections. What I suggest is partly an interpretation of the appearance seen in cleared preparations, partly an assumption. If the buccal tube is to be functional for the only purpose which it can possibly serve, that of a sucking tube, any means of communication between the

buccal cavity and the pumping-pharynx, other than through its lumen, must be cut off. There can be no such communication ventrally, as the tube arises from the floor of the stomatodaeum. But a wide channel exists dorsal to the buccal tube, which is closed, as far as I can determine, by a pair of elastic folds, which lie at the base of the tube, and which pass obliquely upwards and backwards. When the two halves of the tube are brought together, the lips of these folds also come together, and shut off the pumping-pharynx from the buccal cavity.

The function of the buccal tube is obviously that of the straw through which we take our lemon-squash. When the buccal cavity is protracted, its anterior end comes into contact with the skin, through which the piercers are already actively engaged in boring, and possibly its sharp chitinous edges also penetrate to some extent. Its lumen is in direct continuity with the pumping-pharynx, into which blood is drawn by the active pumping action of the dilatator muscles. From the appearance in cleared preparations, a pair of ventral folds lying at the proximal ends of the pieces of the buccal tube would appear to act as valves closing its lumen during relaxation of the dilatators, but I am not able to make out the relations of these in sections.

The piercer-sheath and piercing apparatus.

The piercer-sheath is an invagination of the floor of the buccal cavity which runs backwards beneath the alimentary canal to the posterior margin of the sub-oesophageal ganglion, where it ends about the tendinous insertions of six muscles, the retractors of the piercing apparatus. In section these muscles appear as two lateral groups of three, one on either side of the middle line. Each group is composed of a dorsal and a ventral muscle nearer the middle line, with a lateral muscle lying outside and between them. Between the dorsal pair of muscles, and extending downwards in the middle line, lies a strand of tissue not of a muscular nature. Tracing these structures from behind forwards, it is found that the ventral pair of muscles is inserted into a pair of hollow chitinous 'tendons,' which run a short distance forwards and then join a thin chitinous plate which lies on the floor of the sheath. This plate runs through about two-thirds of the whole length of the sheath, the anterior third being separated by a suture from the posterior two-thirds. In dissections of potash-cleared specimens this posterior plate has broad punctate or granulate margins. The lateral pair also are inserted into tendons, which soon unite to give rise to a hollow chitinous structure, the *ventral piercer*, through the base of which a strand of muscle runs, but which, for the greater part of its length, is simply hollow chitin, apparently without cellular elements. It

carries a chitinous groove, the wall of which is much thickened, along its mid-dorsal line, and the thin flange-like margins, which are more or less folded up about the salivary duct and dorsal piercer, are crenate (Plate VII, Fig. 2). Its appearance in transverse section is shown in the various text-figures. At its anterior end it suddenly becomes much narrower, and terminates in a bilobed, minutely denticulate, boring apparatus. Precise description of the component parts of the piercing apparatus is almost impossible, as, although the length of the apparatus as isolated from a potashed preparation is ·622 mm., the width at the piercing apex is only ·008 mm.

The tendons of the dorsal pair of muscles run along the sides of a short quadrilateral plate, from which runs forward the dorsal piercer, consisting of two thin curved plates, joined by their contiguous edges, and which ends anteriorly in a bilobed denticulate tip, similar to the ventral piercer. A strand of tissue runs along the ventral side of the dorsal piercer for some distance, beneath which is suspended the chitinous salivary duct (Fig. 6). The latter, as far as I can determine, is the true salivary duct, but I have not been able to make out the connection with absolute certainty. This chitinous portion, in any case, takes its origin in the strand of tissue lying between the dorsal pair of retractor muscles runs forward for some distance attached to the underside of the dorsal piercer, then lies free in the groove on the upper face of the ventral piercer. It ends a little short of the anterior extremities of the last-mentioned structures.

The whole of this piercing apparatus enters the buccal tube through the ventral fissure, and is here bent at a definite angle. Into the piercer-sheath opens a pair of tubular glands, described by Pawlowsky (Plate VII, Fig. 1, *p.gl.*). These have a simple epithelial lining of small cells, very much smaller than any other gland cells in the body. Pawlowsky suggests that they may serve to lubricate the piercing organ, but I should think, from the small size of the cells and the absence of any sign of glandular activity in their protoplasm, that they are practically functionless.

The musculature of the piercing apparatus comprises the three pairs of retractors already described, and a pair of large muscles which run from the postero-lateral region of the head on either side beneath the sheath, and are inserted into the tendons of a pair of small muscles rising ventro-laterally from the head-wall beneath the pumping-pharynx. These are the *musculi digastrici* of Pawlowsky, and probably serve, as he suggests, to draw the sheath with its contained apparatus forward, though it is a little difficult to understand just how this result is achieved.

The action of this complex piercer is simply to make a wound. It is inserted by the action of the digastric muscles, aided possibly by differential action of the dorsal and lateral retractors; while by

the relaxation of the digastric muscles and contraction of all three pairs of retractors it is returned to its normal resting position.

Method of feeding.

The method of feeding may now be briefly recapitulated. When the louse has selected a spot, the haustellum is protruded by protraction of the buccal plate, and a superficial fixation effected by the buccal teeth. The same action brings the buccal tube into contact with the skin, and the contained piercing apparatus enters the tissues of the host, and penetrates to the level at which blood is reached. Salivary secretion accompanies the piercer, and possibly contains an anti-coagulin and also has some solvent action. The pumping-pharynx commences its rhythmical dilatations, by means of which blood is drawn in through the buccal tube, and passed backwards to the pharynx, where, by means of alternate contraction of the sphincters and dilatation by the dilatators, it is pumped backwards into the oesophagus, by the peristaltic action of which it is carried to the stomach.

At the close of the feeding process, the piercer is drawn in by its own retractors, the retractors of the labium serving to pull the sheath back. The retractors of the buccal cavity draw in the haustellum and buccal tube.

The fundamental error in Schiödte's classic experiment of cutting off the anterior part of the head of a louse during feeding is that it severs the piercer from its retractors, and it is thus shown as protruding for a distance which is impossible in view of the short 'pitch' of these muscles. The second error was, of course, the assumption that this 'tube' was the channel through which the blood passed.

Interpretation.

In the absence of ontogenetic work proving beyond doubt the homologies of the various structures described, any interpretation will depend primarily on the view taken of the affinities of the Anoplura. Enderlein (1904), being satisfied that these insects are hemipterous, has been able to homologise the parts contained within the piercer-sheath as labium, hypopharynx, and maxillae, corresponding to those of the Rhynchota. Cholodkowsky (1904), on the other hand, states definitely from embryological study, that mandibles and maxillae disappear, and that the piercer-sheath and its apparatus are formed from the labium alone. He expresses his conviction that the Mallophaga and Anoplura are closely related, and includes them in a new order, Pseudorhynchota, an unnecessary proceeding seeing that several names, e.g. Anoplura Leach and Parasita Latreille, already exist for a combination of the two groups. As the Mallophaga are undoubtedly of orthopterous

origin. these two views cannot be reconciled, and one must, perforce, take one side or the other.

I am wholly with Cholodkowsky, whose view is not new, nor solely his own, but has been arrived at independently by Melnikow (1869), Börner (1904), Handlirsch (1903), and is supported by Fulmek (1907), Mjöberg (1910) and myself (1915). I propose to show, not only that the Mallophaga and Anoplura are beyond all question closely related, but that the Anoplura are distinctly nearer, in a great many features, to the Ischnocera, of the two Mallophagan sub-orders.

The antennae of Anoplura have 3—5 articles, as have the Ischnocera, and are filiform, and borne laterally upon the sides of the head. In the Amblycera, the antennae are capitate, with 4 or 5 articles, and are hidden in a groove beneath the head. In all three groups a sensory organ of precisely the same structure is carried in the same position distally upon the ultimate and penultimate articles.

The legs of the three groups are similar in form, and differ considerably from those of most insects. The Amblycera have two equal divaricate claws upon a two-jointed tarsus, except in the Gyropidae, which have the claws remarkably modified. The Ischnocera are generally stated to have a two-jointed tarsus and two claws, except in the Trichodectidae, which have a single claw. This statement is quite wrong. The Ischnocera have only a single joint in the tarsus (though in some species of *Trichodectes* there is an indication of a second) and have in all cases *only one functional claw*. The so-called second claw was undoubtedly a claw to begin with, but is no longer connected with the flexing apparatus, is incapable of voluntary movement, and is, in many species of *Philopterus*, much reduced in size, while in *Trichodectes* it has disappeared. A remarkable intermediate condition occurs in the Anopluran *Haematomyzus proboscideus*, parasitic upon elephants, in which this degraded claw may be seen in transition from its terminal position to the preaxial place of a tarsal hair. The Anoplura in general have a single tarsal joint, and a single claw.

In the Ischnocera meso- and metathorax are always fused; in the Amblycera a distinct mesothorax is present in many genera; in the Anoplura the thorax is quite undivided, except in *Haematomyzus*, which has a prothorax separated off.

The abdomen consists in all three groups of nine segments in the adult, with evidence of at least two more in the larval stages (Harrison, 1915, p. 122). That of the Anoplura resembles certain Amblycera, rather than the Ischnoceran type.

The alimentary canal is generally similar in all three. In the Ischnocera the crop is an asymmetrical diverticulum of the oesophagus. In the Amblycera there is a symmetrical dilatation of

the oesophagus in front of the mid-gut. The Anoplura have neither crop nor proventriculus. In all three a pair of lateral caeca from the mid-gut project forward into the thorax, the malpighian tubes are four in number, and the arrangement and proportions of the hinder parts of the intestine are identical. There are two pairs of salivary glands, and, in the Ischnocera and Anoplura, groups of specialised binucleate cells, richly tracheated, lie about the ducts of these, at the base of the oesophagus.

The respiratory system I have previously shown to be practically identical in all three groups (1915); there being one thoracic and six abdominal pairs of stigmata, two main tracheal trunks, and a system of four transverse commissures in connection with the four main nerve masses. In all Anoplura, and in some Mallophaga of both sub-orders, the lateral trunks are connected by a posterior commissure. I have shown for Mallophaga, and Enderlein (1904) for Anoplura, that the abdominal stigmata, whatever their apparent position, are morphologically upon segments 3—8.

In all three groups the heart is a round sac, situated dorsally in the eighth abdominal segment, with two or three pairs of ostia dorsally in Mallophaga (Fulmek, 1905), three being the number that I find in *Pediculus*. The heart is supported by three bundles of alary muscles on either side, in close relation with which are a group of half-a-dozen pericardial cells. The fat-body is similar in character and distribution, and in the larva is distributed in metameric pockets, each surrounding a little group of calcosphaerites.

The nervous system in all three groups is identical, comprising a brain, sub-oesophageal ganglion, and three thoracic ganglia almost fused together. From the postero-lateral angles of the metathoracic ganglion a bundle of long fibres is given off to the viscera, there being neither nerve cords nor ganglionic enlargements in the abdomen. The nerve supply to eyes, tactile hairs, and other sensory organs, is identical.

The reproductive system shows a still more remarkable similarity. In male Amblycera there are three pairs of testes, those of either side being evenly spaced upon the corresponding vas deferens. The latter open into a paired vesicula, showing more or less fusion, but in most cases completely fused, from which a common ejaculatory duct leads to an eversible sac in connection with the chitinous genitalia. The basal plate of the Amblycera is in most families of the sub-order only a chitinous rod, tapering anteriorly. In the Boopidae and some others, however, a true basal plate is developed. In Ischnocera and Anoplura the testes are four in number, the pair of either side being placed base to base, the corresponding vas deferens arising between the closely apposed bases. The vesicula and ejaculatory duct have the same relations as in the Amblycera, but the vesicula of *Pediculus* is only fused at

its base, the distal three-quarters being free. The chitinous parts of the Ischnocera and Anoplura are also identical in structure; with basal plate, parameres, endomeres, telomeres and penis. *Pediculus* has simpler genitalia, but is not typical of the Anoplura.

With the female, there are ten ovarian tubules, five on either side in Ischnocera and Anoplura, four of these being rudimentary in many Amblycera. These exhibit an identical histological structure, so much so that Gross (1906, p. 378) states that he could not distinguish between them in sections. The oviducts of either side unite in a uterus, into which open cement glands for attaching the egg to its substratum. In the Ischnocera, and in all Anoplura save *Pediculus*, a receptaculum of remarkable structure opens into this uterus by a long narrow duct, the entry of the duct into the receptaculum being marked by a conspicuous chitinous ring.

There is a further correspondence between Ischnocera and Anoplura in the secondary sexual characters, and method of copulation. In both groups, contrary to what obtains in the Amblycera, and in insects in general, the male crawls under the female, grasping her hind legs with his front legs, and flexes the abdomen upwards to insert the penis. In both groups secondary clasping appendages are developed upon the antennae in many genera, and these occur always on the third article.

These comparisons, which have embraced all the systems of organs, quite convince me, not only that the Mallophaga and Anoplura are very closely related, but also that the Anoplura are distinctly more Ischnoceran than Amblyceran in character. In this connection it is interesting to note that, while Amblycera occur normally upon marsupials, neither Ischnocera nor Anoplura are found upon these lower mammals. The general organisation that I have outlined also discounts hemipterous affinities for the Anoplura, as the Rhynchota differ markedly in every detail of structure.

I consider, then, that the close relationship between Mallophaga and Anoplura may be taken as definitely established, and that in any attempt to explain the mouth-parts of the latter, those of the former must be taken into account. The Mallophaga are of orthopterous origin, and are most nearly related to the Copeognatha, which have fairly typical orthopterous mouth-parts. These parts have undergone successive reduction in the two Mallophagan sub-orders. The Amblycera have mandibles, a maxilla consisting of a basal piece formed of fused cardo and stipes, with a four-jointed palp, and an undivided endopodite. (A chitinous splint homologous with the Psocid "fork" occurs in most Amblycera. If this belong to the maxilla, a thing of which I am personally not convinced, then what I have called an undivided endopodite would consist of galea only.) The labium is reduced, and consists of

well-defined sub-mentum and mentum, the latter with rudimentary palps. In the Ischnocera, the mandibles are strongly developed, but the maxilla is reduced to a conical lobe, and the labium is only rarely separable into sub-mentum and mentum. In both sub-orders, as also in the Copeognatha, there is present a remarkable hypopharynx, which consists, in its typical form, of a lyriform chitinous sclerite lying in the floor of the mouth, with a pair of sclerosed glands lying below and behind it, from which tracheated ducts run forwards and upwards, turn backwards and unite, the common duct entering the sclerite. Running backwards from the glands are two chitinous pedicels, which are tendinous in nature, and serve for the attachment of muscles. These structures, which would appear to have lost whatever function they once possessed, have been studied by Cummings (1913), who has shown that they can undergo very remarkable variation and specialisation. I have studied most of the types used by Cummings, and can confirm his work.

As regards previous interpretations of the Anopluran mouth-parts, we have, as already stated, that of Enderlein (1904), who finds labium, hypopharynx, and maxillae in the piercer-sheath; and that of Cholodkowsky (1903), who states that the embryonal mandibles and maxillae disappear, and that the whole piercing apparatus is formed from the labium, which becomes internal in position owing to a fold of the ventral integument of the head growing forward over it. Enderlein's account is based entirely on his belief that the Anoplura are Rhynchota, and he does not advance one tittle of evidence in support of his views that the structures which he names labium, hypopharynx, and maxilla are really homologous with the same structures of other insects. His bare statement, and a vague reference to comparative anatomy, are apparently expected to convince us. But an examination of the rest of his comparative description does not serve to justify any unreserved acceptance of his statements. His ventral lamella of the pharynx is the buccal plate, the roof of the buccal cavity. His 'fulturae' are the diverging cornua of the same plate. His 'dorsal lamella' is the floor of the pumping-pharynx. His larynx—and if terminology must be borrowed from vertebrate anatomy, it might at least be applied to analogous structures—is the pharynx. He indicates the 'mandibles' as projecting freely into the buccal cavity in a figure (1905, p. 634, f. 4) which he certainly has the grace to call 'stark schematisiert,' whereas these structures lie in the body-cavity, and have no connection with the alimentary canal. Enderlein has accurately described and figured the several structures lying in the piercer-sheath, as seen in dissection, but all the rest of his account, as well as his conclusions, may be dismissed without further consideration.

Cholodkowsky, however, cannot be so lightly dealt with. He examined *Pediculus* embryos, both entire, and in sections, and he states quite definitely that the mandibular and maxillary rudiments fuse and disappear, that the labium alone forms the piercing apparatus, and that all other chitinous plates and splints appearing about the mouth are secondary structures. Apart from this general statement he attempts no detailed interpretation. Until Cholodkowsky's ontogenetic work is either confirmed or refuted by renewed embryological investigation, the broad facts must be accepted.

The following suggestions are on this account quite speculative in character, but are based on a considerable knowledge of Mallophagan mouth-parts. I assume that the close affinity between Mallophaga and Anoplura is sufficiently well proven. I suggest that the Anoplura show stronger points of resemblance to the Ischnocera than to the Amblycera. As there is no evidence as to the degree of relationship between the two Mallophagan sub-orders, the fact that the Anoplura favour one or the other is possibly not of very vital importance, if the two sub-orders are branches of a single stock. But it is possible that these sub-orders, which do differ markedly in certain respects, have branched off at different times from an original Psocid stock, in which case the resemblance of the Anoplura would have more significance. For our present purpose, however, the significance lies in the fact that the maxilla has almost completely disappeared in the Ischnocera, a fact which agrees with Cholodkowsky's statement of what happens with *Pediculus*. I am not prepared to believe that the mandible disappears, for I think that the structures which Enderlein describes as mandibles in *Haematopinus suis* are reduced mandibles. The German author is certainly wrong in believing that they project into the buccal cavity, and he is again wrong in his homology for these structures in *Pediculus*, as what he calls mandible here is simply an optical section of a chitinous thickening passing almost completely round the head. But mandible-like structures do exist in the first larva of *Pediculus* (Fig. 7), and

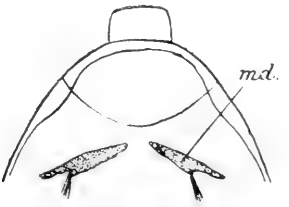

Fig. 7. Anterior portion of head of larva, to show mandibles.

have the same general relations as those described by Enderlein for *Haematopinus*. Mjöberg (1910, p. 205) figures them also for *Antarctophthirus*, in which they are conspicuous even in the adult. In the later larval stages of *Pediculus* these mandibles gradually disappear. If these structures be not mandibles, as the double muscular attachments would suggest, they are difficult to account for. They are certainly functionless. Against such an interpretation is the fact that they lie in the haemocoele. Here Cholodkowsky's account of the growth forward of a fold of the ventral integument to cover the labium, which is drawn inwards in the invagination of the piercer-sheath, suggests an explanation. The mandibles may also be involved in this process of epiboly, and, by the subsequent absorption of the upper wall of the fold, become internal. An intermediate condition actually occurs in the Mallophagan genus *Philandesia*, in which the mandibles are almost completely shut off by two lateral folds of the ventral surface of the head.

In any case the mandible and maxilla may be dismissed as far as any participation in the actual mouth-parts of the Anoplura is concerned. Cholodkowsky states that the piercing apparatus is formed entirely from the labium. As the labium in Mallophaga is reduced to a sub-mentum and mentum, with rudimentary, one-jointed palps, it is impossible to believe that all the structures involved could be derived from labium alone. I suggest that the two plates lying in the floor of the piercer-sheath may represent the sub-mentum and mentum.

The buccal tube and the piercing apparatus are thus left unaccounted for. I believe that the whole of these structures have been derived from modifications of the Mallophagan hypopharynx. The figures of Cummings (1913) show the remarkable fluidity of this structure and the variation it has undergone even in closely allied species. The piercer-sheath is a depression in the floor of the buccal cavity, into which opens a pair of apparently functionless glands, as well as the salivary duct, in all of which it agrees with the Mallophagan hypopharynx. Moreover this suggestion affords a means of homologising the buccal tube, which otherwise is difficult to account for. It is comparatively easy to picture the two halves of this tube as the anterior cornua of the hypopharynx, which lie above, and lateral to, the central depressed portion; and also to imagine the latter drawn backwards into the long piercer-sheath, this invagination involving the associated parts. It would be idle, in view of the speculative nature of the suggestion put forward, to attempt to press the homologies still further, and to try and indicate the precise origin of the dorsal and ventral piercers.

The buccal plate and teeth do not demand any particular

explanation. The former is simply the thickened cuticular lining of the buccal cavity. The latter may be modifications of the ordinary labial hairs, or may be quite independent structures. Lice are capable of an infinite variety of processes of their chitinous cuticle.

I have examined for comparison the mouth-parts of several other genera of Anoplura, though not in section; and I find them all to be of the same general type, with minor differences in detail.

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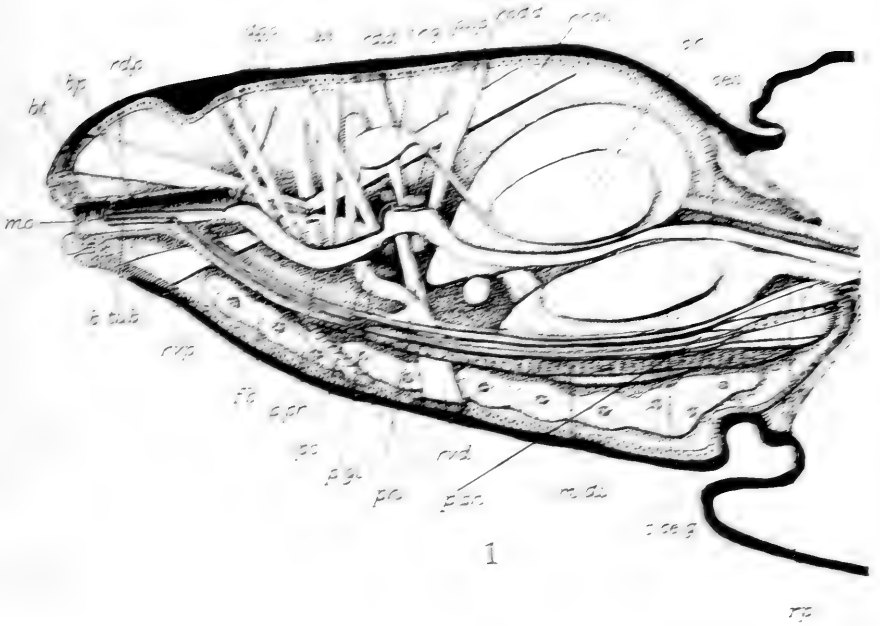
Explanation of Plate VII.

Fig. 1. Slightly schematised reconstruction of the right half of the head of *Pediculus humanus*, viewed in the sagittal plane.

Fig. 2. The piercing apparatus.

Explanation of Lettering for Text-figures and Plate.

as. anterior pharyngeal sphincter; *bc.* buccal cavity; *bp.* buccal plate; *bt.* buccal teeth; *b.tub.* buccal tube; *br.* brain; *div.* piercer diverticulum; *dp.* dorsal piercer; *dpp.* dilatators of pumping-pharynx; *f.b.* fat-body; *fr.g.* frontal ganglion; *la.* ? labium; *lcbp.* left cornu of buccal plate; *ldp.* left dorsal protractor of buccal cavity; *lvd.* left ventral dilatator of pharynx; *lvp.* left ventral protractor of buccal cavity; *md.* mandible; *m.di.* musculus digastricus; *mo.* mouth; *oes.* oesophagus; *p.gl.* Pawlowsky's gland; *ph.* pharynx; *p.ph.* pumping-pharynx; *ps.* pharyngeal strainer; *p.sh.* piercer-sheath; *p.sp.* posterior pharyngeal sphincter; *rcbp.* right cornu of buccal plate; *rdd.* right dorsal dilatator of pharynx; *rdp.* right dorsal protractor of buccal cavity; *rp.* retractors of piercers; *rpdd.* right posterior dorsal dilatator of pharynx; *rrbc.* right retractor of buccal cavity; *rvd.* right ventral dilatator of pharynx; *rvp.* right ventral protractor of buccal cavity; *sd.* ? salivary duct; *s.oe.g.* sub-oesophageal ganglion; *vp.* ventral piercer.



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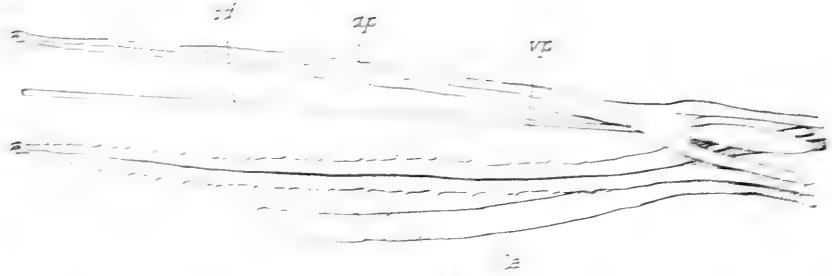

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On Some Gynandromorphic Specimens of Abraxas grossulariata.

By L. Doncaster, Sc.D., *Fellow of King's College.*

[*Read* 21 February 1916.]

In the spring of 1915 I bred two abnormal specimens of *Abraxas grossulariata* among those that I was rearing for work on the determination of sex. Each of them combines to a certain extent the characters of both sexes, and they may therefore be described as gynandromorphic, although they are not lateral gynandromorphs of the type in which one half of the insect has the characters of one sex and the other half of the other sex. The first specimen (reference no. of family 14.13) hatched on March 18; it appeared externally to be a male, and was killed and recorded as a male before I noticed that it was an exception to the rule of sex-limited inheritance that a *grossulariata* female mated to a *lacticolor* male gives all male offspring *grossulariata*. The remainder of the family consisted of 24 *lacticolor* females, in accordance with the rule that all females from such parentage should be *lacticolor*, and this particular family belonged to a strain which produces only, or almost exclusively, females in certain families (see *Journ. of Genet.* III, 1913, p. 1 and IV, 1914, p. 1). Superficially the moth appeared a typical *lacticolor* male, but when I discovered that it was an exception to the normal rule of sex-limited inheritance, I examined it more closely, and dissected its abdomen. The examination showed that the left antenna was less strongly pectinated and slightly shorter than the right; the left wings somewhat smaller and the left side of the body with smaller spots. Internally the testes were white instead of orange, and apparently empty; no other abnormalities in the internal genitalia were noticed, but the moth at the time of dissection was not in a good state of preservation for detailed observation. I sent the moth, with the next specimen to be described, to Mr F. N. Pierce, a leading expert on the external genitalia of British moths, and from his description, which I summarize below, it will be seen that while on the right side the genital armature is nearly like that of a normal male, on the left in addition to malformed male organs there are portions of female structures.

The main features of Mr Pierce's report are (1) the right antenna is male, the left female; the frenulum of the left wing is of the male type and well developed, that of the right, male but imperfect. In the external genitalia the chief points are that

the uncus, anus and ovipositor are each divided; the right valva is not unlike that of a normal male; the left is abnormal and has attached to it a second anus and half of the ovipositor. The uncus is divided, and its left half bears on its outer side the second half of the ovipositor. The aedeagus is normal; on the left side of the eighth segment there is a portion of the ostium bursae.

The chief peculiarities of the specimen are thus (1) that though predominantly male, it has the *lacticolor* character which from its parentage should be confined to females; (2) throughout the body the right side is male, the left imperfectly developed or tending towards the female type, with division of parts of the genital armature which are normally median. The internal genital organs were as far as is known imperfectly developed male organs.

The second specimen (family 14.31), hatched on April 9, attracted my attention at once, since I could not decide at all when I examined it alive whether it was male or female. I attempted to pair it with both sexes, but it neither showed sexual instincts nor attracted males. In pattern it is intermediate between *grossulariata* and *lacticolor* (much like the Rev. J. M. Woodlock's variety 'Q,' cf. *Journ. of Genet.* v, No. 3, 1916); the black spots are much reduced, but it must be regarded as being *grossulariata* from their shape, and also because they are well developed on the lower surface of the wing. As the parents were *grossulariata* female and *lacticolor* male the *grossulariata* character should appear only in the male offspring. The remainder of the family consisted of 11 *grossulariata* males, 11 *lacticolor* females. On dissection of the abdomen, the internal genitalia were poorly developed female organs; there was an ovary on each side, with four egg-tubes on the left and two or three on the right; some of the follicles (egg-tubes) contained a few eggs, about four of which were fairly well developed. There was a bursa copulatrix and I found no trace of testes. The main points from Mr Pierce's report are (1) the antennae are male in character, as are the frenula and retinacula of the wings. The external genitalia are asymmetrical, very imperfect, but largely male, with only small traces of ovipositor, but with the external armature of the bursa on the eighth segment and no trace of aedeagus. The valvae are largely aborted, especially that on the right side. The uncus is divided, and the right half bears spines characteristic of the ovipositor. The eighth segment is largely female in character, and bears a fairly well developed ostium bursae.

The important features of this specimen are therefore (1) the predominantly male character of its external organs, both genital armature, which, however, is very imperfectly developed, and secondary sexual characters, combined with the pattern which should accompany the male sex from such parentage, though this

was badly developed; (2) the internal genital organs were, however, ovaries, and contained a few quite well developed eggs.

In consequence of the discovery of these gynandromorphs, one, if not both, of them associated with an apparent failure of the normal sex-limited transmission, it occurred to me that possibly the two exceptional females of family 12.25 described in *Journ. of Genet.* IV, 1914, p. 15, might also have been to some extent gynandromorphic. Both paired with males, and one laid eggs, but these proved infertile. I therefore sent the specimens to Mr Pierce, and he reported as follows. The specimen which laid no eggs had distinctly asymmetrical genitalia, the right side being on the whole better developed, while the left, although female in character, had all the organs more or less malformed. In the specimen which laid about 20 infertile eggs, the external genitalia were asymmetrical and imperfect, and Mr Pierce is of opinion that certain structures may perhaps represent rudiments of male organs.

These facts place in a somewhat new light the conclusions with regard to exceptions to the normal sex-limited inheritance arrived at in the paper referred to (*Journ. of Genet.* IV, 1914). It was there concluded that failure of the normal sex-limited transmission of characters might involve sterility, and support was given to this idea from the frequent sterility of the tortoise-shell tom-cat (*Proc. Camb. Phil. Soc.* XVII, p. 307 and *Journ. of Genet.* V, 1915, p. 65). It now appears, however, that in two cases of apparent failure of sex-limited transmission (if the specimen in family 14.31, which contained ovaries, is so regarded), the individual is in reality a gynandromorph, and that the sex-limited characters are normal as regards one part of the sex of the specimen. And further, in both the cases previously described as exceptions to sex-limited transmission (family 12.25), although the whole insect was predominantly female, yet on one side the external genitalia were abnormal, and in one case showed indications of male structure. It is possible, therefore, that both these insects were to a very slight extent gynandromorphic, and that the *grossulariata* pattern, where *lacticolor* was to be expected, is connected with the possession of a certain amount of male tendency in individuals which were preponderatingly female. If such a view be adopted, it would mean that hitherto no true exceptions to normal sex-limited inheritance are known in *Abraxas grossulariata.*

PROCEEDINGS AT THE MEETINGS HELD DURING THE SESSION 1915—1916.

ANNUAL GENERAL MEETING.

October 25, 1915.

In the Comparative Anatomy Lecture Room.

PROFESSOR NEWALL, PRESIDENT, IN THE CHAIR.

The following were elected Officers for the ensuing year:

President:

Prof. Newall.

Vice-Presidents:

Dr Shipley.
Dr Fenton.
Prof. Eddington.

Treasurer:

Prof. Hobson.

Secretaries:

Mr A. Wood.
Dr Arber.
Mr G. H. Hardy.

Other Members of the Council:

Mr F. J. M. Stratton.
Prof. Woodhead.
Mr C. Forster Cooper.
Mr C. E. Inglis.
Dr Duckworth.
Dr Crowther.
Mr H. H. Brindley.
Mr H. Hamshaw Thomas.
Dr Bromwich.
Dr Doncaster.
Mr C. G. Lamb.
Dr Marr.
Mr J. E. Purvis.

The following were elected Associates of the Society:

E. Lindsay Ince, Trinity College.
Hon. H. Onslow, Trinity College.
A. R. McLeod, Gonville and Caius College.

The following Communications were made:

1. Examples illustrating the use of Integral forms. By R. Hargreaves, M.A., St John's College.

2. On certain arithmetical functions. By S. Ramanujan. (Communicated by Mr G. H. Hardy.)

3. Vector Integral Equations and Gibbs' Dyadics. By C. E. Weatherburn. (Communicated by Mr G. H. Hardy.)

4. A self recording electrometer for Atmospheric Electricity. By W. A. D. Rudge, M.A., St John's College.

5. The resolution of asymmetric quinquevalent nitrogen compounds. By J. Reilly. (Communicated by Professor Pope.)

November 22, 1915.

In the Sedgwick Museum.

Professor Newall, President, in the Chair.

The following were elected Fellows of the Society:

C. E. Weatherburn, M.A., Trinity College.
H. Jeffreys, B.A., St John's College.

The following Communications were made:

1. Notes on Oysters: recent and fossil. By Prof. Hughes.

2. Fossil Zones and Geological Time. By Dr Marr.

3. (1) On induced herpetomoniasis in Birds.
   (2) Notes on certain Protozoa which may be found in cases of Dysentery from the Mediterranean War Zone.

By Dr H. B. Fantham and Miss Annie Porter.

4. On a little-known concealed coalfield in Oxfordshire. By Dr Arber.

February 21, 1916.

In the Advanced Zoology Lecture Room.

PROFESSOR NEWALL, PRESIDENT, IN THE CHAIR.

The following was elected a Fellow of the Society:

A. W. R. Roberts, M.A., Trinity College.

The following Communications were made:

1. On some gynandromorphs in *Abraxas grossulariata*. By Dr DONCASTER.

2. A preliminary account of the structure of the mouth-parts in the Body-louse. By L. HARRISON. (Communicated by Professor Nuttall.)

3. The Field and the Cordon of a Plane Set of Points. By E. H. NEVILLE, M.A., Trinity College. (Communicated by Mr G. H. Hardy.)

May 22, 1916.

In the Botany School.

PROFESSOR NEWALL, PRESIDENT, IN THE CHAIR.

The following were elected Fellows of the Society:

S. Chapman, M.A., Trinity College.
H. B. Milner, B.A., Trinity College.
E. H. Neville, M.A., Trinity College.
H. T. J. Norton, M.A., Trinity College.

The following Communications were made:

1. Some considerations on the geographical distribution of species. By Dr WILLIS. (Communicated by Dr Arber.)

2. A preliminary note on the internal structure of *Pityostrobus* (*Pinites*) *macrocephalus* from the Lower Eocene. By C. P. DUTT, B.A., Queens' College. (Communicated by Professor Seward.)

INDEX TO THE PROCEEDINGS

with references to the Transactions.

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